PHY 121 Mechanics *S.G. Rajeev*

Lecture 19

Equilibrium and Elasticity

•If the distance between two points on a body remains fixed in time, it is a *rigid body* .

•A rigid body has six degrees of freedom: it can move as a whole or it can turn around any of three axes.

•The motion as a whole is described by the position of the center of mass. If the sum of all the forces acting on a body is zero, its momentum will be a constant. Its center of mass will move with constant velocity.

•If the sum of the torques acting on a rigid body is zero, its angular momentum will be a constant.

•A rigid body is in *equilibrium* if the sum of forces acting on the body is zero *and* the sum of the torques is zero.

•A body in equilibrium is not necessarily at rest: its momentum and angular momentum will be constant.

•But if it is at rest forever (i.e., it has zero velocity and angular velocity forever) then it must be in equilibrium.

Example

A uniform ladder of length L and mass m leans against a (frictionless) wall at an angle θ to the horizontal. The ground has a coefficient of static friction of μ_s . How far along the length of the ladder can a person of mass M climb up before the ladder will slip?

Solution

•Let the length she can climb up the ladder before it slips be l .

•The weight of the ladder acts at its center of gravity, which is a distance $\frac{L}{2}$ along the ladder.

•There is a vertical force F_y at

the point where the ladder touches the ground. There is a horizontal frictional force of $F_x = \mu_s F_y$ at the same point.

•There is a horizontal force f at the point where the ladder touches the wall; there is no vertical force there because the wall is frictionless.

From the requirement that forces must add up to zero,

$$N = (m+M)g$$

and

$$f + F_x = 0$$

Thus

$$f = -\mu_s(m+M)g.$$

We need more information. The

torque of the weight of the ladder from the point of contact at the ground is

$$m\frac{L}{2}g\cos\theta$$

and that of the person is

$$Mlg\cos\theta.$$

This is cancelled by the torque of the force f :

$$\mu_s(m+M)gL\sin\theta = g\cos\theta[\frac{mL}{2} + Ml]$$

This can be solved for l .

Elasticity

●Most solids are of course not exactly rigid. They stretch a little if we pull on them. But if the force is not too strong, they will recover their shape once they are released. This is the phenomenon of *elasticity*.

● *Stress* at a point is the force per unit area at that point. *Strain* is the change in position of the point relative relative to the neutral position; i.e., the change in the length divided by the original length.

●For small stresses, the strain is proportional to the stress. The ratio stress over strain is called

the modulus of elasticity. Also called Young's modulus.

Example

Steel has modulus $2 \times 10^{11} Nm^{-2}$. Bone has $9Nm^{-2}$.