PHY 121 Mechanics *S.G. Rajeev*

Lecture 2

Motion in a Straight Line

•To make any progress in science we must begin by studying simple *models* for natural systems. In mechanics we begin by studying the motion of a single *particle* in a *straight line*.

•A *particle* is an indivisible object; either it is so small that we don't have to think of how it is made up, or, all its parts move together as one unit. If you drop a melon from a tower, it moves as a *particle* until it hits the ground. After it splatters, we can no longer think of it as a single particle.

The Position Co-ordinate

•It is convenient to think of points on the straight line as labeled by numbers. Imagine the line is a highway and we start measuring the distance starting from the exit to your hometown. This point is the *origin*. Any point on the line has a unique number attached to it, the distance in meters (say) from the origin. This number is the *co-ordinate*. The *origin* itself has co-ordinate *zero*.

•By convention the line is drawn horizontally and the co–ordinates of points to the *right* of the origin are considered *positive* while those to the *left* are *negative*.

Position as a Function of Time

•The motion of a particle on the line is known if you know where it is at any given time. *Time* can also be represented by a number t , the number of seconds after some chosen initial event (e.g., when you started trip on the highway.) t is *positive* for instants *after* the start and *negative* for those *before* the start.

•If you know the co-ordinate of the particle at each time, you know everything about the motion of the particle. At least, everything can be deduced from it. Thus the motion of a particle on a line is determined by a function $x(t)$ which assigns to each value of the real variable t (time) another real number $x(t)$ (the co–ordinate at time t).

•Now suppose we know this function; it can be represented as a graph, or as a table.

Average Velocity

•Suppose we know the position $x_1 = x(t_1)$ at some time t_1 and $x_2 = x(t_2)$ at some later time t_2. The *average velocity* of the particle in the interval $[t_1, t_2]$ is $\frac{x_2 - x_1}{t_2 - t_1}$. It is the same as the slope of the line connecting (t_2, x_2) to (t_1, x_1).

•What is the velocity $v(t)$ at the instant t? Imagine a really small interval $[t, t + \Delta t]$ containing t. (Δ is a Greek letter usually denoting a small change in some quantity.) Suppose the displacement during that time interval is $\Delta x(t) = x(t+\Delta t) - x(t)$. If Δt is small enough, we will get a good approximation to $v(t)$: $\frac{\Delta x}{\Delta t}$. As Δt becomes smaller

and smaller this becomes closer and closer to $v(t)$, the velocity at time t. Mathematically,

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x(t)}{\Delta t}$$

•This is just the concept of *derivative* in *Calculus*.

$$v(t) = \frac{dx(t)}{dt}$$

• *Example:* Let the position of a particle be given by the function

$$x(t) = a + bt + ct^2$$

where

$$a, b, c$$

are constants. Its velocity at time t is

$$v(t) = b + 2ct.$$

Note that it is *NOT*

$$b + ct$$

and *NOT* even

$$\frac{a + bt + ct^2}{t}.$$

Speed vs. Velocity

• *Speed* is the magnitude of *velocity*. If you look at the speedometer of your car you can tell your speed, but you must look out the windshield to know your direction. *Velocity* is a *vector*: it has a *direction* as well as a *magnitude*.

Analysis in Mechanics

- You must have heard that there are functions which are *continuous* but *not differentiable*. The laws of physics forbid such functions from arising in mechanics. *The position of a particle as a function of time will always be a differentiable function*. Indeed it is even a smooth function: all its derivatives exist.
- Does anyone know what a real analytic function is? The position of a particle as a function of time is smooth but is usually not real analytic.

Acceleration

- *Acceleration* is the *derivative of velocity* with respect to time. In other words *acceleration is the second derivative of position* with respect to time.
- *Example*

If

$$x(t) = x_0 + ut$$

(where x_0, u are constants), the velocity is a contant:

$$\frac{dx}{dt} = u$$

and the acceleration is zero:

$$\frac{d^2x}{dt^2} = \frac{du}{dt} = 0.$$

Example
If

$$x(t) = x_0 + ut + \frac{1}{2}at^2$$

the *acceleration* is a *constant*. (Again x_0, u, a are constants. For,the velocity $v(t)$ at time t is,

$$\frac{dx}{dt} = x_0 + u + at$$

and the acceleration is just

$$\frac{d^2x}{dt^2} = \frac{dv(t)}{dt} = a.$$

Example
If

$$x(t) = \frac{2}{t^2} - \frac{4}{t}$$

we have the acceleration at time t is just $a(t)$:

$$a(t) = \frac{d^2x}{dt^2} = \frac{d}{dt}(-\frac{4}{t^3} + \frac{4}{t^2})$$

and

$$a(t) = \frac{12}{t^4} - \frac{8}{t^3}.$$

- Thus if the acceleration is a constant, the position is a quadratic function of time.

$$x(t) = x_0 + ut + \frac{1}{2}at^2$$

What is the physical meaning of the constants x_0, u, a ? a is obviously the constant acceleration. x_0 is just the position at $t = 0$. And u is just the initial velocity.

The Principle of Equivalence

•An object that is falling freely is a simple example of motion with constant acceleration. (If it falls for too long, we can no longer neglect wind resistance.) The acceleration due to gravity at the earth's surface is roughly equal to $9.8\ m/s^2$ in magnitude.

•In the absence of wind resistance, *all bodies fall at the same rate no matter what their composition or mass*. This is actually a basic principle of modern physics, the *principle of equivalence*.

Exercise

A boy stands on the bridge over Route 390 and drops a stone which hits a car that is moving at a uniform velocity of u . If the *height* of the bridge is h what was the distance D of the car from the point of impact when the stone was dropped?

Solution

Let g be the acceleration due to gravity and t the time for which the stone fell. We will ignore the effects of air friction and wind resistance. The initial velocity of the stone is zero. Let us measure vertical distance from the point

where the stone is dropped. Then,

$$h = \frac{1}{2}gt^2, \quad D = ut$$

so that

$$t = \sqrt(\frac{2h}{g})$$

and

$$D = u\sqrt(\frac{2h}{g}).$$

If $u = 90\ km/hr$ and $h = 10\ m$ we have, (knowing that $g = 9.8m/s^2$)

$$D = \frac{90 \times 10^3}{3600}\sqrt(\frac{2 \times 10}{9.8})\ m$$

or

$$D = 35.7\ m.$$

Exercise

A train travelling at a speed u must be stopped with a constant acceleration in a distance D . What is the acceleration a required?

Solution

$$0 = u + at, \quad D = ut + \frac{1}{2}at^2$$

so that

$$D = -\frac{1}{2}at^2 = -\frac{1}{2}\frac{u^2}{a}$$

and

$$a = -\frac{u^2}{2D}.$$

The required acceleration is of course *negative*.