PHY 121 Mechanics *S.G. Rajeev*

Lecture 20

Fluid Dynamics

•A *fluid* is a substance that can flow. In particular, it has no shape; its shape is determined by the container it is in. A fluid is not a rigid body.

•Liquids and gases are fluids. Gases are compressible; i.e., their density can be made to increase by exerting pressure on them. Liquids are almost incompressible. Of course with a sufficiently large pressure even a liquid can be made to change its density but not by much.

•Even some things that look like solids (e.g. glass or rock) can best

be thought of as very slowly flowing liquids.

•The molecules that make up a solid oscillate (vibrate) around a mean position. The molecules that make up a fluid move around everywhere at random, with no fixed average position. This is why fluids behave so differently from solids.

•The *density* of any substance is its mass per unit volume. *Pressure* is the force per unit area exerted by the fluid.

• *Pressure* is also the potential energy per unit volume contained in the fluid. To see this imagine that we have a fluid of uniform pressure p in a cylindrical container of

length L and cross-sectional area A.

•Imagine that one side is closed off by a movable piston. The force acting on the piston is

$$F = pA.$$

If we move the piston over a distance ΔL, the work we do is

$$\Delta W = F\Delta L.$$

The change of volume is

$$\Delta V = A\Delta L.$$

Thus the work done on the fluid (the amount by which its potential energy has increased) is

$$\Delta W = p\Delta V.$$

This shows that pressure is the derivative of potential energy with respect

to volume.

Hydrostatics

•Consider a fluid (like water) of uniform density ρ at the surface of the Earth. The pressure increases as we go down deep under the surface.

•In fact we can see that the pressure p at a depth h is

$$p = p_0 + \rho g h$$

where p_0 is the pressure at the surface.

•To see this consider a part of the fluid of cros-sectional area A and depth between y_1 and y_2 . Let p_1 and p_2 be the pressures at the top and bottom, respectively. The force at the top is $p_1 A$ and that at the bottom of this part is $p_2 A$

. The weight of the fluid contained in the volume is

$$\rho A(y_2 - y_1)g.$$

•To be in equilibrium, the force due to the weight must equal in magnitude to the force due to the difference in pressures:

$$\rho g A\{y_2 - y_2\} = p_2 A - p_1 A$$

so that

$$p_2 = p_1 + \rho g(y_2 - y_1).$$

•Thus pressure increases linearly with the depth. We have assumed that the density is a constant in this argument. For the Earth's atmosphere this is not quite correct;

the density of the atmosphere decreases as the pressure falls at an altitude of several miles. The pressure still deceases with height but not linearly any more.

•The atmospheric pressure at the Earth's surface is $1.013 \times 10^5 N m^{-2}$. The SI unit of pressure is *Pascal*. That is, one Pascal is one Newton per square meter.

Archimedes Principle

•Due to the difference of pressures at the top and the bottom, any body submerged in a fluid will be pushed upwards by the fluid: this is the force of *buoyancy*.

•What is the force of buoyancy acting on a body submerged in a liquid? If the body is a cylinder of height h the answer is easily found: the force at the top is $p_1 A$ and that at the bottom is $p_2 A$. So the force is $(p_2 - p_1)A$. This is just $\rho g A h$ where ρ is the density of the *fluid*. *Not the density of the submerged body.*

•For any body no matter what its shape, we have the Archimedes principle:

The force of buoyancy acting on a body fully or partially submerged in a fluid is the weight of the fluid it displaces.

•Archimedes was probably the first physicist. But he was so far ahead of his time that no one understood what he was doing until many centuries later.

•Ships, submarines and lighter-than-air balloons operate on this principle.

Hydrodynamics

•If fluids are not in equilibrium, the pressure and velocity are related to each other by conservation of energy: pressure is a form of potential energy which can be converted to the kinetic energy of fluid motion.

•Imagine a small cell of fluid of volume ΔV that has constant density ρ and is at pressure p, has velocity v and height h. Its total energy is the sum of the potential energy due to pressure, gravitational potential energy and kinetic energy:

$$p\Delta V + \rho \Delta V g h + \frac{1}{2}\rho \Delta V v^2.$$

•At any other point this has to be

equal to the total energy. Thus we have Bernoulli's principle:

$$p + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}.$$

•We have ignored loss of energy due to friction. There is a partiular kind of friction called *viscosity* which cause fluids to lose energy. We have ignored it so we are dealing with what is called an *ideal fluid*. Feynman once said that an ideal fluid is "dry water". We have also assumed that the fluid has constant density; i.e, that it is incompressible.

•Airplanes work on the Bernoulli principle. The air on top of the wings has further to go so it moves

faster. Hence it has lower pressure. It is this pressure difference that supports the weight of the airplane.