PHY 121 Mechanics *S.G. Rajeev*

Lecture 22

Resonance and Waves

Resonance

• We saw that a particle in Simple Harmonic Oscillation of angular frequency ω has acceleration proportional to the displacement:

$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

• The restoring force is thus $-m\omega^2 x$ where m is the mass.

• What if there is an additional *driving* force $F(t)$ acting on the particle? We would have

$$\frac{d^2x}{dt^2} = -\omega^2 x + \frac{F(t)}{m}.$$

This is a *differential equation.*

• Don't get scared if you have never solved a differential equation. I will tell you what the solution is in the case of interest to us.

• The most interesting case is when the driving force is also a periodic function of time. If you push a kid on a swing each time he swings next to you, you are exerting a periodic driving force.

• Suppose that the driving force is a cosine function

$$F(t) = F_0 \cos(\Omega t)$$

for some angular frequency Ω. This is the frequency of the force, not the same as the natural angular frequency ω of the oscillation.

• F_0 is some constant describing the maximum force.

• Then we can see that

$$x(t) = B\cos(\Omega t)$$

is a solution to our differential equation, with a properly chosen constant B.

• Indeed,

$$\frac{d^2x}{dt^2} = -B\Omega^2\cos(\Omega t)$$

so that

$$-B\Omega^2\cos(\Omega t) = \omega^2 B\cos(\Omega t) + \frac{F_0}{m}\cos(\Omega t)$$

• The cosine cancels out giving

$$B(\omega^2 - \Omega^2) = \frac{F_0}{m}$$

or,

$$B = \frac{F_0}{m(\omega^2 - \Omega^2)}.$$

• This says that no matter what its natural frequency ω is, an oscillator will move at the frequency of the driving force. The amplitude of this induced oscillation is B.

•. Something strange happens as the frequency of the driving force Ω approaches the natural frequency ω. The amplitude of the oscillation grows, and is *infinite* as the two frequencies coincide.

• Of course in reality B will not become infinite. Effects we have ignored such as friction will come into play for large oscillations and the true value of B is a large but finite number.

• But the moral of the story is still valid: *a driving force with the same frequency as the natural frequency of an oscillator will produce large oscillations.*

• This is the phenomenon of *resonance.*

• Opera singers can make a wine glass shatter by singing at exactly the natural frequency of the glass: the induced oscillations are so large that the glass doesn't have the strength to withstand them.

Waves

• *Waves* are caused by the collective oscillation of a large number of particles.

• The most familar waves are the waves on the surface of water: each particle of water bobs up and down but the position of maximum displacement moves with time. A similar effect is produced at basketball games by fans standing up and then sitting down one row at a time.

• Waves are periodic in both space and time. The simplest kind of wave is a *sinusoidal* wave:

$$\phi(x,t) = A\sin(kx - \omega t).$$

A is the *amplitude* of the wave, and ω is its *angular frequency.*

k is called the *angular wave number.*

• $\phi(x,t)$ describes the displacement of a particle in mean position x at time t. Notice that if we hold x fixed (i.e., watch what happens at a fixed location) and vary t, we just get simple harmonic motion. Each particle is executing simple harmnic motion with angular frequency ω.

• Hence the period of the wave is $T = \frac{2\pi}{\omega}$.

• Now vary x keeping t fixed (i.e., take a snapshot of the wave). Now we have a function periodic in position. The distance between two successive peaks is called the

wavelength usually denoted by λ. We have

$$\lambda = \frac{2\pi}{k}.$$

Wavelength is the spatial analogue of period.

• Notice that the wave only depends on the combination $kx - \omega t$. If you watch a spot with a fixed displacement (e.g., a peak) it will appear to move at a constant velocity

$$v = \frac{\omega}{k}.$$

This is the velocity of the wave.

• Note that this has nothing to do with the velocity of the partciles that are bobbing up and down. This is the speed with which the peak of the wave moves.

•In fact the oscillating particles don't even need to be moving in the same direction as the wave. They could be moving perpendicular to the direction of the wave, (*transverse oscillations*) as in the case of water waves. Or they could be moving in the same direction as the wave (*longitudinal waves*)as in a Slinky.

•Light is a form of transverse wave. Sound is a longitudinal wave, due to oscillations of the particles of air.