PHY 121 Mechanics *S.G. Rajeev*

Lecture 23

Sound Waves

- Vibrations of air is *sound.* These vibrations cause the displacement of air molecules from their average positions.
- For a wave travelling to the right, the displacement from the mean position of an air molecule is

$$s(x,t) = s_0 \cos(kx - \omega t).$$

- These vibrations also cause small changes in the pressure and density of air.
- The velocity of sound in air (at $20°C$) is $343\ m/s$. Audible sound has a frequency between $20\ Hz$

and $25,000\ Hz$.

Decibel Scale

- The *intensity* of sound waves is the average amount of energy crossing a unit area normal to the direction of propagation.
- For the above traveling wave, it is

$$I = \frac{1}{2}\rho v \omega^2 s_0^2$$

where ρ is the density of air and v the velocity of sound. The derivation is in the textbook.

- It is common to measure this intensity on a logarithmic scale:

$$\beta = \log \frac{I}{I_0}, \qquad I_0 = 10^{-12} W m^{-2}.$$

The base of the logarithm is *ten* and not e .

- The standard intensity (with $\beta = 0$) is chosen to be barely audible to human ears.
- The unit of sound intensity is called a *Bel.* In fact it is more comon to use a tenth of a *Bel* or *decibel.*
- A sound wave of $20 dB$ has an intensity I given by

$$2 = \log \frac{I}{I_0}$$

or

$$I = I_0 \times 10^2$$

which is a *hundred* times what the human ear can barely hear.

- At $120\ dB$ your ears will start to hurt. Anything above that is dangerous to your ear.

Standing Waves

•A wave

$$\phi(x,t) = A\sin(kx - \omega t)$$

moves to the right: the direction of increasing x.

•The following wave moves to the left:

$$\phi(x,t) = A\sin(kx + \omega t).$$

•If we add the two, we produce an interesting effect: we get an phenomenon called *interference*, so that the two waves combine to produce a *standing wave*.

•Recall a little identity from trigonometry:

$$\sin(a+b) + \sin(a-b) = 2\sin a\cos b.$$

•The sum of the waves moving in oppsoite directions is the *standing wave*

$$\phi = 2A\sin(kx)\sin(\omega t).$$

It is called that because the position of a peak or trough is fixed. For example, the wave vanishes at $x = 0$ for all time.

•If we tie a string down at two ends and pluck, the kind of wave that can travel on it must have the property that the wave vanishes at the end points.

•Thus it can be a standing wave, whose wavelength is the *twice* the length of the string: then the string will contain a half a period

of a sinewave with zeros at the endpoints.

•It can also be a standing wave whose wavelength is the twice length of the string divided by an integer: in this case the string contains several periods of the sine wave. The wavelengths are allowed to be

$$\lambda_n = \frac{2L}{n}, \quad \text{for } n = 1, 2, 3, \cdots.$$

•The velocity of the wave is independent of its length. It depends only on the tension on the string T and its mass per unit length μ :

$$v = \sqrt{(\frac{T}{\mu})}$$

•We will not derive this fact here. See the textbook for a related discussion.

•Thus the frequencies allowed are

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}$$

•Thus the frequncies allowed are multiples of a *fundmental frequency*.

•Even our vocal chords operate on this principle. Trained singers can produce a single note: sound of a single frequency. But usually a string produces sound which has many frequencies, all multiples of its fundamental frequency.