PHY 121 Mechanics *S.G. Rajeev*

Lecture 7

The Use of Force

Force is a Vector

•Let $\mathbf{F}_A$ and $\mathbf{F}_B$ be the forces that A and B exert on C . What is the total force on C due to A and B ?. It is just the vector sum of the two forces:

$$\mathbf{F} = \mathbf{F}_A + \mathbf{F}_B$$

•Conversely it may be useful to decompose a given force into its components in various directions. Most often we will choose two or three perpendicular directions and find the component of force in each of these directions.

Exercise

A block of mass $8.4\ kg$ is pulled along a horizontal frictionless table by a cord that exerts a force of $43.0\ N$ at an angle of $32.4°$ above the horizontal. What is the acceleration of the block?

Solution

Let m be the mass of the block, $\mathbf{F}$ the force due to the cord. Also, let θ be the angle that $\mathbf{F}$ makes with the horizontal. The force can be split into its vertical and horizontal components:

$$\mathbf{F} = F\cos\theta\mathbf{i} + F\sin\theta\mathbf{j}.$$

Now the vertical component of the

force plus the weight of the block is exactly cancelled by the force that the table exerts on the block. Thus the net force acting on the block is just the horizontal component of $\mathbf{F}$:

$$\mathbf{F}_{\text{tot}} = F\cos\theta\mathbf{j}.$$

The acceleration is horizontal and has magnitude

$$a = \frac{F\cos\theta}{m}$$

or

$$a = \frac{43.0 \times \cos(32.4°)}{8.4}\ ms^{-2} = 4.32\ ms^{-2}$$

Forces on an Airplane

Exercise

To change the direction of an airplane, its wings are banked at an *angle* θ . Assume that the magnitude of the lift produced by the airplane is such that the plane *does not have any vertical acceleration*. Also, that its *speed* remains constant at v . What is the *radius* of the circle on which the plane is turning?

Solution

The wing must produce a greater lift as the plane turns; if the lift remains unchanged as you bank

the wings, the plane will accelerate downward.
Let F be the magnitude of the lift produced by the wings. The vertical component of lift is balanced by the weight:

$$F\cos\theta = mg.$$

The horizontal component provides the centripetal acceleration:

$$F\sin\theta = m\frac{v^2}{R}$$

where R is the radius of the circle on which the airplane is turning. Taking ratio of the two equations,

$$\tan\theta = \frac{v^2}{gR}$$

or

$$R = \frac{v^2}{g}\cot\theta$$

If $v = 640.0\ km\ hr^{-1}$ and $\theta = 45°$ we have

$$\begin{aligned} v &= 640.0 \times 10^3 \times (3600)^{-1}\ ms^{-1} \\ &= 178\ ms^{-1} \end{aligned}$$

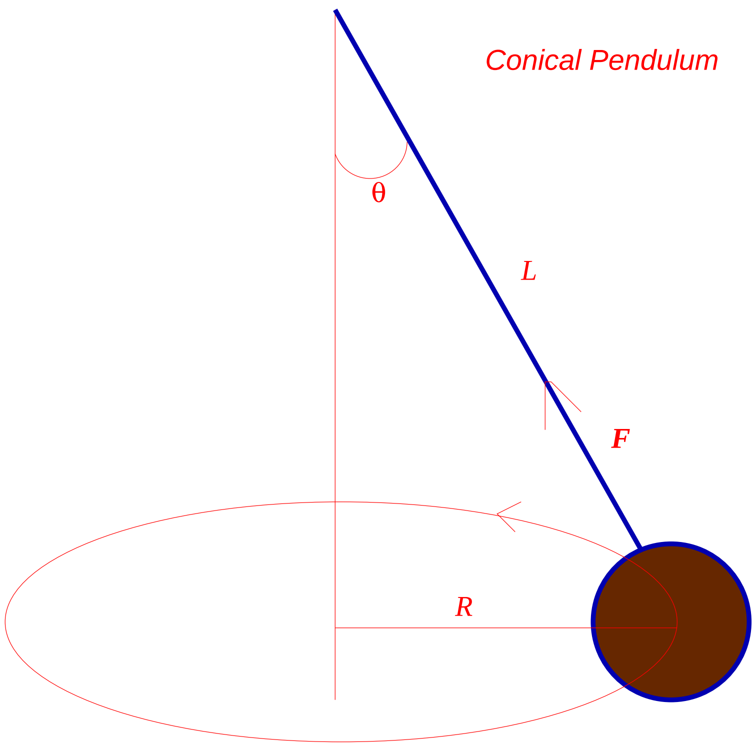
and

$$\begin{aligned} R &= (178)^2 (9.8)^{-1} \times 1\ m \\ &= 3220\ m = 3.22\ km. \end{aligned}$$

Conical Pendulum

Exercise

A mass m is hung from the ceiling by a massless of string of length L. It is made to move in a circle at constant speed, with the string making an angle of θ to the vertical. Find the period of rotation.



Solution

The force of the mass due to gravity is pointed straight downward. Let T be the tension on the string. The vertical component of the tension is cancelled by the weight:

$$T\cos\theta = mg$$

while the horizontal component is the centripetal force:

$$T\sin\theta = mR\omega^2.$$

Here R is the radius of the circle and ω the angular velocity. Taking ratios

$$\tan\theta = \frac{R\omega^2}{g}.$$

Now, from the diagram, we see that L ,

$$R = L\sin\theta$$

so that

$$\omega^2 = \frac{g}{L\cos\theta}$$

But,

$$T = \frac{2\pi}{\omega}$$

where T is the period of revolution. Thus

$$T = 2\pi\sqrt{(}\frac{L\cos\theta}{g}).$$

In fact $L\cos\theta$ is the vertical distance of the mass from the point of suspension. Thus no matter what the value of the length and angle are, *two pendulums that hang at the same height from the support will have the same period.*

Also, the period of a pendulum is independent of the mass m .