PHY 121 Mechanics *S.G. Rajeev*

Lecture 9

Kinetic Energy

•The *kinetic energy* of a particle of mass $m$ and velocity $\mathbf{v}$ is

$$K = \frac{1}{2}m|\mathbf{v}|^2.$$

This is just one kind of *energy*; we will see other forms of energy soon.

•The *SI* unit of energy is *Joule*. In other words,

$$1\ J = 1\ kg\ m^2\ s^{-2} = 1\ Nm.$$

In *cgs* units the unit of energy is *erg*. Thus

$$\begin{aligned} 1\ J &= 1\ kg\ m^2\ s^{-2} \\ &= 10^3 g(100)^2\ cm\ s^{-2} \\ &= 10^7\ ergs \end{aligned}$$



• *One Joule is ten million ergs*. So it is important to know if you are using *SI* or *cgs* units.

•The word *energy*, like *work* or *force* or *momentum* has a special technical meaning in physics.



Work

•A particle of mass $m$ moves on a line. It starts from an initial point $x_{\text{in}}$ to a final point $x_{\text{fi}}$ under the influence of a *constant* force $F$. By how much has its kinetic energy changed?

•Let $v_{\text{in}}$ be the initial velocity, and $v_{\text{fi}}$ the final velocity. We have

$$x_{\text{fi}} = x_{\text{in}} + v_{\text{in}}t + \frac{1}{2}at^2$$

where $a$ is the constant acceleration. Also,

$$v_{\text{fi}} = v_{\text{fi}} + at$$

We can eliminate $t$ to get

$$x_{\text{fi}} - x_{\text{in}} = v_{\text{in}}\frac{v_{\text{fi}} - v_{\text{fi}}}{a} + \frac{1}{2}a(\frac{v_{\text{fi}} - v_{\text{in}}}{a})^2$$



or,

$$x_{\text{fi}} - x_{\text{in}} = \frac{v_{\text{fi}}^2 - v_{\text{in}}^2}{2a}.$$

Thus

$$v_{\text{fi}} - v_{\text{in}}^2 = 2a(x_{\text{fi}} - x_{\text{in}}).$$

•The initial kinetic energy is $\frac{1}{2}mv_{\text{in}}^2$ and the final kinetic energy is $\frac{1}{2}mv_{\text{fi}}^2$ so that the change in kinetic energy is

$$\begin{aligned} \Delta K &= \frac{1}{2}mv_{\text{fi}}^2 - \frac{1}{2}mv_{\text{in}}^2 = ma(x_{\text{fi}} - x_{\text{in}}) \\ &= F(x_{\text{fi}} - x_{\text{in}}). \end{aligned}$$

•Thus we see that at least for motion in a straight line with constant acceleration, *the change in the kinetic energy is the product of* *force by the displacement.*

•This product of force and displacement is called *work*. Thus work is the amount of energy you have added to a particle by exerting the force on it. *Work can be negative*; for example if you are making a particle stop, you have done a negative amount of work on it.

•What if the force is not a constant? Assume that the motion is still on a straight line. Also suppose first that the force is constant $F_1$ from $x_{\text{in}}$ to some point $x_1$ ; then another constant $F_2$ from $x_1$ to $x_{\text{fi}}$ . The total energy that has been added to the system in the



first distance is $F_1(x_1 - x_{\text{in}})$ then we add $F_2(x_{\text{fi}} - x_1)$ . The total work done is the sum

$$F_1(x_1 - x_{\text{in}}) + F_2(x_{\text{fi}} - x_1)$$

•In general force will be a function of position; how hard the particle is being pushed will depend on where it is. How will we find the total work done in going from $x_{\text{in}}$ to $x_{\text{fi}}$ ? As with many other problems in physics, the trick is to split up a hard problem into many pieces, each piece being simple.

•Let us split up the interval $(x_{\text{in}}, x_{\text{fi}})$ into small pieces of length $\Delta x$ each. If the size of each interval is small enough, we can assume that the



force will not change by much within that interval: we can take it to be the average value there. So we know how much work is done in each interval. Then we get the total work done by adding up all the individual pieces.

•Thus

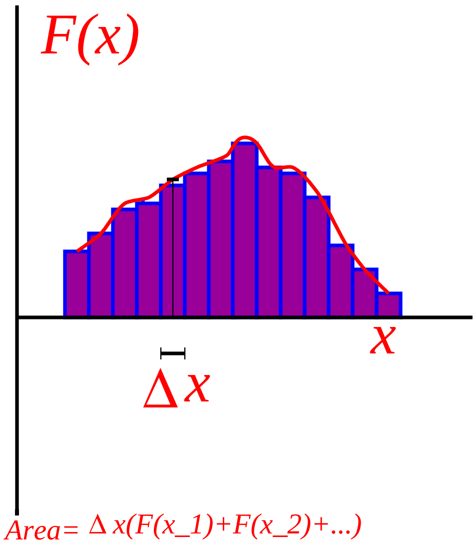
$$W = (F_1 + F_2 + F_3 \cdots)\Delta x$$

where $F_1$ is the average value of force in the first interval, $F_2$ the average value in the second interval and so on. Of course, if $\Delta x$ is very small, there will be many terms in the sum. In the limit as $\Delta x$ goes to zero there will be an infinite number of terms in the sum. Still, the product of this large num-



ber and the small number $\Delta x$ will tend to a finite answer: the *integral* of force with respect to distance.

•This can be viewed as the area under the curve when $F(x)$ is plotted as a function of $x$ . ( The part of the area under the $x$ -axis is counted as negative.) See diagram.




•The total work done is thus

$$W = \int_{x_{\rm in}}^{x_{\rm fi}} F(x)dx$$



Hooke's Law

•A spring is a simple device for producing a force. The more you stretch it, the greater the force. Indeed there is a law of Hooke:

*The force produced by a spring is proportional to the distance.*

•As an equation, if $x$ is the length by which the spring is stretched,

$$F(x) = -kx$$

where $k$ is a positive constant. This is a property of the spring and is called its (what else?) *spring constant*. Note that if $x$ is positive we are stretched. The force will tend to push you in the opposite direction, which is why there is



a negative sign in the above equation.

•Suppose we make a particle move from a point $x_{\rm in}$ to a point $x_{\rm fi}$ under the influence of a spring of spring constant $k$. What is the work done? If you do the integral you will get

$$\int_{x_{\rm in}}^{x_{\rm fi}}(-kx)dx = \frac{1}{2}k(x_{\rm in}^2 - x_{\rm fi}^2)$$