S10Symmetries
Problem Set 2
Due Feb 17 2010

(1) Recall that $su(3)$ is the Lie algebra of anti-hermitean 3×3 matrices. What is its dimension? (i.e., the number of independent basis elements).What is the rank of this Lie algebra? (i.e., the maximum number of commuting basis elements) Find a basis for $su(3)$. (**Hint** Gell-Mann matrices play a role here analogous to Pauli matrices for spin.) Find a positive quadratic form that is invariant, analogous to L^2 for $so(3)$.

(2) For a given magnitude of angular momentum L, and principal moments of inertia $A_1 < A_2 < A_3$, what is the maximum rotational kinetic energy allowed for a rigid body? What is the minimum energy?

(3) Solve the Euler equations in terms of the Jacobi functions using the ansatz

$$L_1 = C_1 \text{cn}(\omega t, k), \ L_2 = C_2 \text{sn}(\omega t, k), \ L_3 = C_3 \text{dn}(\omega t, k)$$

Determine how C_1, C_2, C_3, ω, k depend on K, A_1, A_2, A_3, L^2.