S10Symmetries
Problem Set 4
Due Mar 24 2010

(1) Let B be the space of complex analytic functions of a single variable z with the norm $< \psi, \psi >= \int |\psi(z)|^2 e^{-z^*z} \frac{d^2z}{\pi}$. Define $a\psi = \frac{\partial \psi}{\partial z}$. Find the adjoint of this operator. Show that together they satisfy $[a, a^\dagger] = 1$. Find the orthonormal basis of eigenvectors of $a^\dagger a$.

(2) Show that $su(n)$ (traceless matrices) is a Lie subalgebra of $u(n)$. Is it an ideal? Find an ideal of $u(n)$ that is orthogonal to $su(n)$ with respect to the invariant inner product $\mathrm{tr} u^\dagger v$.

(3) Find the dimension and rank of $su(4)$. Find a basis of its Cartan subalgebra. What are the simultaneous eigenvectors of the commutators with the Cartan generators? (i.e., find the analogues of the raising and lowering operators). Find the 15 matrices that are analogues for $su(4)$ to the Pauli and Gell-Mann matrices.