GRAVITATION F10

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Lecture 26

1. Radiation Reaction

1.1. **The power radiated by an electromagnetic source is proportional to the square of the second time derivative of the dipole moment.** The total charge cannot be time dependent. So the first moment that can depend on time is the dipole. At large distances the field produced by other moments can be ignored.

1.2. **The Larmor formula gives the power radiated by a non-relativistic an accelerated charged particle.**

$$P = \frac{2}{3}\left[\frac{e^2}{4\pi\epsilon_0}\right]\frac{a^2}{c^2}$$

where a is the acceletaion and e the charge.

1.3. **The loss of energy and momentum due to radiation exerts a force on the particle, called radiation reaction.** The derivation of proper equations of motion including this radiation reaction turned out to be an enormously difficult problem. The correct answer was established only recently (2000) although the problem itself goes back to the nineteenth century. The non-relativistic approximation is much more tractable

1.4. **The equation of motion of a charged particle in an electrostatic potental including radiation reaction is now well-establishe.**

$$\frac{d}{dt}\left[\mathbf{v} + \epsilon\nabla U\right] = -\nabla U$$

where U is $\frac{e}{m}$ times the eletrostatic potential and $\epsilon = \frac{2}{3mc^3}\left[\frac{e^2}{4\pi\epsilon_0}\right]$. When U is the Coulomb potential this describes a particle falling into the center of the potential.

1.5. **The emission of gravitational waves is much harder to understand due to nonlinearities of Einstein's equations.** There has been much recent progress in the numerical solution of GR. In particular, spectacular results on the orbits of two black holes spiralling into each other have been obtained by Pretorius and by Campanelli et. al.

1.6. **In the Post-Newtonian approximation simple results can be obtained.** If all the particles are moving at velocities small compared to light, the radiation emitted is small. This is tractable by analytic methods.

1.7. **The power of gravitational radiation emitted is proportional to the square of the third derivative of the quadrupole moment.** Now you see why it is so small in the Post-Newtonian limit. To detect gravitational radiation directly (LIGO, LISA etc. experiments) we will need to look for much stronger sources which are outside this approximation.

$$P = \frac{G_N}{45c^5} \dddot{D}_{\alpha\beta}^2$$

This is the analogue of the Larmor formula.

1.8. **A rotating neutron star has been found to slow down at the rate predicted by this formula.** These results of Hulse and Taylor were the first, indirect, detection of gravitational radiation.