# GRAVITATION F10

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## 1. Problem Set 2 Due Oct 13 2010

1.1. **Consider the metric $ds^2 = \frac{dx^2+dy^2}{y^2}$ on the upper half plane.**

1.1.1. *Show that the transformation $x+iy \equiv z \mapsto \frac{az+b}{cz+d}$ is a symmetry of the metric if the parameters $a,b,c,d$ are real numbers with $ad-bc \neq 0$..*

1.1.2. *Solve the geodesic equation of this metric. (It will be useful to exploit conservation laws.)*

1.2. **Assume that the metric of space-time in a weak gravitational field $\frac{|\phi|}{c^2} << 1$ is $ds^2 \approx \left[1+\frac{2\phi(x)}{c^2}\right] c^2dt^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$. Use the variational principle to derive the equation of motion of particles, in the approximation that the velocity is small compared to $c$. Compare to the equations of Newtonian gravity.**

1.3. **A particle of mass $m$ and charge $q$ is moving in a gravitational field given by the metric $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ as well as an electromagnetic field with potential $A_\mu$. What is the action principle of this system? What are its equations of motion?**