Gravitation F10

S. G. Rajeev

October 13, 2010

1 Problem Set 3 Due Oct 27 2010

1.1 Consider again the Poincare metric $ds^2 = \frac{dx^2+dy^2}{y^2}$ on the upper half plane

1.1.1 Show that the curvature tensor has only one independent component in two dimensions. Find this component for this metric.

1.1.2 Get the equation for the infinitesimal deviation between two nearby geodesics in this metric. What is the significance of the sign of the curvature?

1.2 Recall that $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

1.2.1 Show the identity $\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0$

1.2.2 Prove the analogous identity (Bianchi) for the curvature tensor $D_\mu R^\alpha_{\nu\rho\sigma} + D_\nu R^\alpha_{\rho\mu\sigma} + D_\rho R^\alpha_{\mu\nu\sigma} = 0$

1.2.3 Find a tensor $G_{\mu\rho}$ (a linear combination of the traces of the Riemann tensor) that satisfies $D^\mu G_{\mu\rho} = 0$.

1.3 In Yang-Mills theory (a generalization of Maxwell's electrodynamics) the potentials A_μ are matrices and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$

1.3.1 Find the analogue of the Bianchi identity

1.3.2 Derive the field equations from the variational principle (in Minowski space) $S = \frac{1}{4}\int \mathrm{tr} F_{\mu\nu}F^{\mu\nu}dx$