GRAVITATION F10

S. G. RAJEEV

1. Problem Set 4 Due Nov 10 2010

1.1. A centrally symmetric metric can be brought to a special form.

$$ds^2 = -dr^2 - r^2\left[d\theta^2 + \sin^2\theta d\phi^2\right] + h(r,t)dt^2 + 2a(r,t)drdt$$

1.1.1. *Find functions $h(r,t), a(r,t)$ such that the metric satisfies the vacuum Einstein's equations.*

1.1.2. *Compute the all the components of the curvature tensor in these co-ordinates. Where are they singular?*

1.1.3. *Solve the equation for radial null geodesics. Identify the event horizon. Show that there is no singularity in the metric at the event horizon.*

1.2. A Killing vector is an infinitesimal transformation that leaves the metric invariant.

1.2.1. *Consider an infinitesimal co-ordinate transformation $x^\mu \mapsto x^\mu + \epsilon v^\mu, \quad |\epsilon| << 1$. Show by direct caluclation that the infinitesimal change in the metric is.*

$$D_\mu v_\nu + D_\nu v_\mu$$

If this quanity vanished,v^μ is called a Killing vector field.

1.2.2. *Find all the Killing vector fields of the Minkowski metric.*

1.2.3. *What are the Killing vector fields of the Schwarschild metric expressed in the co-ordinates of the previous problem?*

1.3. There is a notion of the commutator of two vector fields.

$$[u,v]^\mu = u^\nu\partial_\nu v^\mu - v^\nu\partial_\nu u^\mu$$

1.3.1. *By writing this in terms of covariant derivatives show that the commutator transforms as a tensor.*

1.3.2. *Show that the commutator satisfies the Jacobi identity.*

$$[[u,v],w]^\mu + [[v,w],u]^\mu + [[w,u],v]^\mu = 0$$

This means that the set of vector fields forms a Lie algebra.

1.3.3. *Is the commutator of two Killing vector fields still a Killing vector field? Prove your answer.*

1.3.4. *Find the commutators of the Killing vector fields of Minkowski space.*

1.3.5. *And of the Schwarschild metric.*