GRAVITATION F10

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1. Problem Set 5 Due Dec 8 2010

1.1. **Show that an observer following a radial time-like geodesic in a Schwarschild metric will fall to the center in finite proper time .** Use Painleve co-ordinates, so that your co-ordinate system extends into the interior of the blackhole.

1.2. **Consider the null geodesics for the Schwarschild metric.**

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

It will be useful to parametrize the orbit by $u_0 = \frac{r_s}{r_0}$, where r_0 the smallest value of r. Choose the co-ordinate system such that this point (the perihelion) corresponds to $\phi = 0$. Derive a differential equation for $u = \frac{r_s}{r}$ along with initial conditions.

1.2.1. *Is it possible for a photon to orbit the center in a circle? If so, what is the radius of this circle?*

1.2.2. *Numerically plot an orbit that has $r_0 >> r_s$ and hence small deflection.*

1.2.3. *Are there orbits that go around the center more than once before going off to infinity? What kind of images would be produced by gravitational lenses if such orbits exist?*

1.3. **Consider a static mass density $\rho(r)$ that is spherically symmetric around some center.** That is, $T_{00} = \rho(r),\ T_{0i} = T_{ij} = 0$.

1.3.1. *Find the solution to Einstein's equations with this source.*

1.3.2. *Verify that your answer reduces to the correct Newtonian limit.* You can use the computation of the Ricci tensor in the last problem set to solve this problem.