Asymptotic Methods in Science and Engineering PHY493(Special Topics in Physics)

Realistic physical problems cannot be solved exactly: physics is based on approximation methods. All the approximation methods of physics produce expansions that are not convergent. So why do they work so well? It turns out that they are asymptotic expansions, which make sense in some pie-shaped region of the complex plane, unlike expansions that converge inside a circle. We will illustrate this with the method of steepest descent, which is also called the method of stationary phase (optics), the WKB method (quantum mechanics) or boundary layer theory (fluid mechanics).We will then apply it to various examples from these physical disciplines. Higher orders in the expansion can be calculated by using Feynman diagrams. We will perform some one and two loop calculations to illustrate the method.

There is no prescribed textbook for the course, but the book Advanced Mathematical Methods for Scientists and Engineers: Asymptotic Methods and Perturbation Theory by C. Bender and A. S. Orszag is recommended for reading.

Syllabus

1. The Stirling expansion of the Gamma Function: why a divergent expansion is more useful than a convergent expansion
2. The method of steepest descent for one dimensional integrals: example of the asymptotic expansion of Bessel functions
3. Asymptotic series; Pade' approximants; continued fractions.
4. The method of stationary phase in optics; Eikonal; Diffraction
5. Steepest Descent for Multi-Dimensional integrals;Feynman Diagrams; Sample one and two-loop calculations.
6. WKB method in quantum mechanics; the anharmonic oscillator.
7. Solution of non-linear differential equations by WKB method; application to fluid mechanics; boundary-layer theory
8. Scalar Quantum Field theory. One and Two loop calculation of the effective potential.
9. Introduction to renormalization theory.