The Franck-Hertz Experiment

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We conduct the classic Franck-Hertz experiment in which electrons are accelerated through a mercury vapor and inelastically collide. We find the energy required to initiate inelastic collisions between electrons and mercury atoms to be 4.69 ± 0.05 eV, in agreement with the lowest excited state of mercury (6^3P_0). The dependence of the Franck-Hertz curve on the tube temperature is explored, and we find a decrease in average peak spacing with increased temperature. We find the measurement of the lowest excited state to be stable with temperature if derived from minima spacings, and inconsistent if derived from the maxima spacings, in agreement with previous studies.

I. INTRODUCTION

James Franck and Gustav Ludwig Hertz, German scientists at the University of Berlin, were the first to demonstrate experimentally the quantized nature of matter in their famous experiment published in the German Physical Society on April 24th, 1914 [1]. They shared the Nobel Prize[1] in 1925 for this "discovery of the laws governing the impact of an electron upon an atom."

The original experiment consisted of measuring the current of electrons that were accelerated through a heated tube containing mercury (Hg) gas. Once the electrons reach a kinetic energy corresponding to the lowest excited state of Hg (the 6^3P_0 state, see Fig 1), inelastic collisions between the electrons and Hg atoms will occur, and the Hg atom will be excited to this state. Electrons accelerated to a kinetic energy corresponding to the next excited state (6^3P_1) will also excite the Hg atoms, however this state is unstable, with a lifetime $\sim 10^5$ times shorter than the 6^3P_0 state [2]. The 6^3P_1 state decays almost immediately back to the ground state by the spontaneous emission of a photon, and is then ready to be excited again. Hence while Hg atoms in the 6^3P_0 state will collide elastically with another incoming electron, with the electron losing a negligible amount of kinetic energy in the process (because of the large mass difference), in the same time period there will be $\sim 10^5$ collisions exciting other Hg atoms to the 6^3P_1 state. Franck and Hertz's original experiment included a window through which the wavelength of the emitted photons could be measured, which they found to be 2536 Å [2].

By measuring the current of electrons exiting the tube of gas as a function of their energy, Franck and Hertz observed dips in the measured current corresponding to the onset of inelastic collisions. While most analyses of the Franck-Hertz experiment assume the current minima occur at integer multiples of the 6^3P_1 excitation energy (4.89 eV, see Fig 1), Rapior, Sengstock, and Baev [3] claim that the minima spacing actually grows with each successive minima. They model this effect by considering the following phenomena: Once an electron reaches sufficient kinetic energy to undergo inelastic collisions (4.67 eV), it will then on average travel one mean-free-path before colliding with a Hg atom. In this time, the electron is further accelerated, gaining more energy and thus possibly exciting one of the higher Hg energy levels (6^3P_1 or 6^3P_2). When the voltage has been sufficiently increased (roughly double the first minima), electrons will be able to undergo two collisions before reaching the grid, causing this effect of added energy to occur twice. The effect of this is a linear growth of the minima separation ΔV with the minima order n:

$$\Delta V(n) = \left(1 + \frac{\ell}{L}(2n-1)\right) V_a, \tag{1}$$

where V_a is the accelerating voltage at which inelastic collisions begin to occur (4.67 V) and L is the distance between the cathode and the grid. See Sec III of [3] for a derivation of Eq 1. The slope of this line allows the determination of the mean free path ℓ, and the intercept allows the determination of the lowest excitation energy $E_a = eV_a$ (e = electron charge). Since the electrons will follow a Maxwell-Boltzmann velocity distribution, their

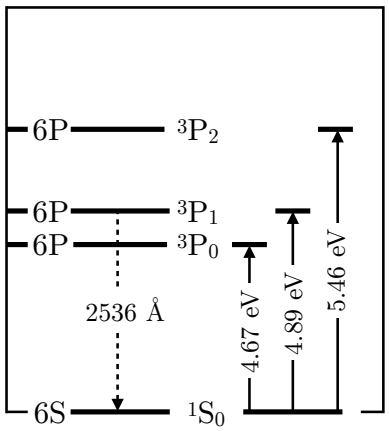

FIG. 1: Energy levels of Hg relevant to this analysis [2, 3].

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[1] https://www.nobelprize.org/nobel_prizes/physics/laureates/1925/

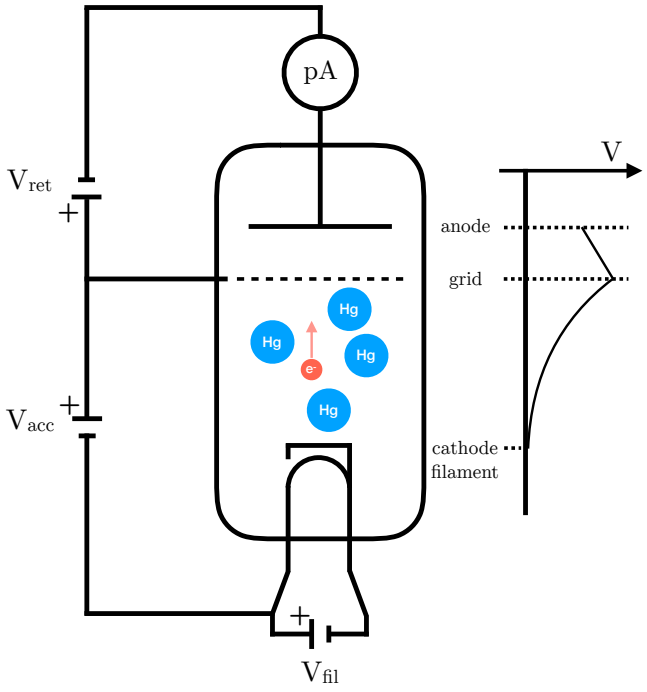

FIG. 2: Schematic of the Franck-Hertz apparatus used.

mean free path ℓ is given by

$$\ell = \frac{1}{\sqrt{2}N\sigma} = \frac{kT}{\sqrt{2}p\sigma} = \frac{kT}{\sqrt{2}p(\pi R_0^2)}, \tag{2}$$

where $R_0 \approx 1.5$ Å is the cross-sectional radius of a Hg atom, and the Hg vapor density N is strongly sensitive to the temperature T of the Hg gas via the ideal gas law $p = NkT$. Thus we expect the mean free path of the electrons, and hence the minima spacings, to vary with the oven temperature as well. Specifically, the average spacing between minima should decrease with increased temperature. In this analysis we consider the dependence of the measured current minima on the oven temperature to explore this effect.

II. EXPERIMENTAL APPARATUS

Fig 2 shows a schematic of our experimental setup. The filament voltage V_{fil} heats a filament resulting in the emission of electrons onto the cathode. These electrons are then accelerated from the cathode to the grid by the electric field created by the accelerating voltage V_{acc}. Electrons that reach the grid but have less energy than that corresponding to the retarding potential V_{ret} are halted and collected by the grid. Those with greater energy are able to overcome the retarding potential and make their way to the anode where they are measured as a current by the Keithley picoammeter (pA in Fig 2).

If an electron moving through the tube happens to have a kinetic energy corresponding to the an energy level of Hg, the electron may collide inelastically with a Hg atom, resulting in the electron no longer having enough energy to overcome the retarding potential. As a result of these collisions, at a particular value of the accelerating voltage (corresponding to electrons with kinetic energy matching that of the Hg excited state), a measurable drop in the current recorded by the picoammeter can be seen. Continuously increasing the accelerating voltage will result in electrons again reaching the required kinetic energy to inelastically collide with a Hg atom, causing a second drop in the measured current. Continuing this process results in repeated dips in the measured current at regular intervals (Fig 4), with the spacing between minima corresponding to the energy required to excite the Hg atoms. This is the famous result of the original Franck-Hertz experiment, which Einstein, after hearing a presentation of the results by Franck at a conference, remarked to be "so lovely it makes you cry" [4].

The density of the Hg gas present in the tube is controlled by the temperature of the oven enclosing the tube. In our analysis, we record measurements at five oven temperatures ranging between 135 °C and 175 °C in 10 degree intervals. At each temperature, a "run" consists of recording the current measured by the picoammeter for accelerating voltages from 3 V to 40 V, in steps of around 0.1 V. Higher oven temperatures correspond to a denser Hg vapor and therefore a smaller mean free path for the electrons, reducing the probability of an electron navigating the thick Hg cloud. We therefore expect to measure smaller currents at higher temperatures, as well as observe a decrease in the slope of the line from Eq 1.

III. DATA COLLECTION

The data are recorded using the Franck-Hertz LabVIEW computer program. The LabVIEW program automates data collection after specifying a filament voltage and start, stop and step-size values to sweep the accelerating voltage. For all runs, we use a filament voltage of $V_{\text{fil}} = 5$ V, a retarding voltage of $V_{\text{ret}} = 1.5$ V, and sweep the accelerating voltage from 3 V to 40 V in steps of 0.1 V. This is done for oven temperatures of $T = 135$ °C , 145 °C , and 175 °C. For $T = 155$ °C and 165 °C, a step size of 0.05 V is used instead. The measurements for each run are displayed in Fig 3. Results for two different tube temperatures are shown in Fig 4.

Note that we can immediately see that the minima locations are not equally spaced across different temperature runs. Fig 4 shows the smoothed data for runs at $T = 155$ °C and $T = 175$ °C. From this it can be clearly seen that the average minima spacing shrinks with increased temperature, as expected. This suggests that conclusions drawn directly from measurements of the average minima spacings are unreliable.

To estimate the uncertainty in our measurement of the

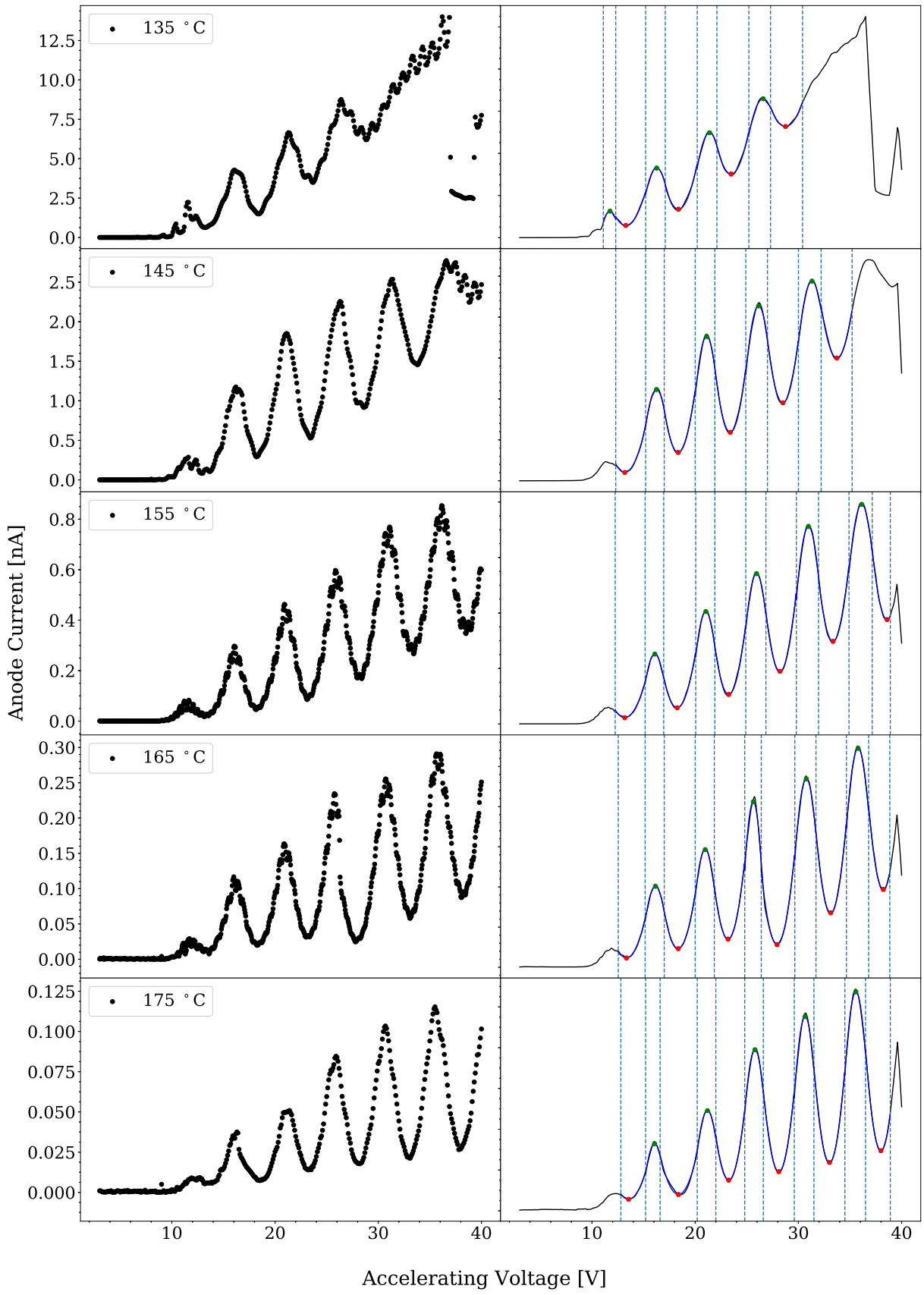

FIG. 3: Data (left) collected for each temperature, and the smoothed and analyzed data (right). The vertical dashed lines correspond to bin edges, and the blue curves are the individual polynomials being fit in each bin. (●) points denote located minima, and (●) points denote located maxima.

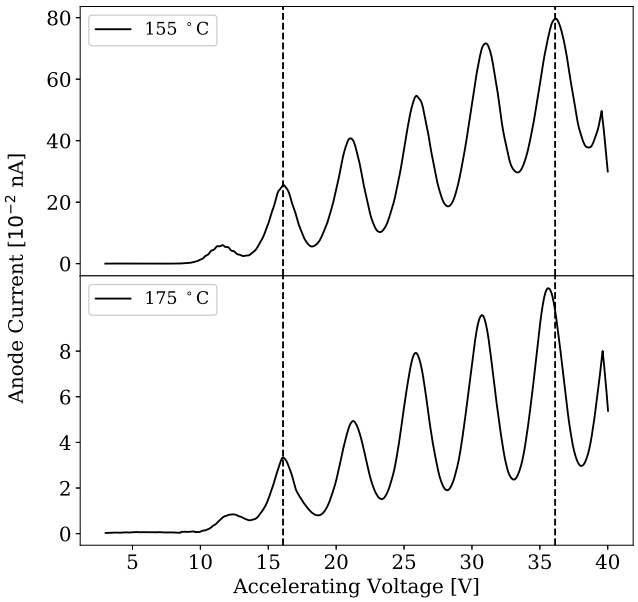

FIG. 4: Franck-Hertz curves for two tube temperatures, shifted horizontally so that the first maxima coincide.

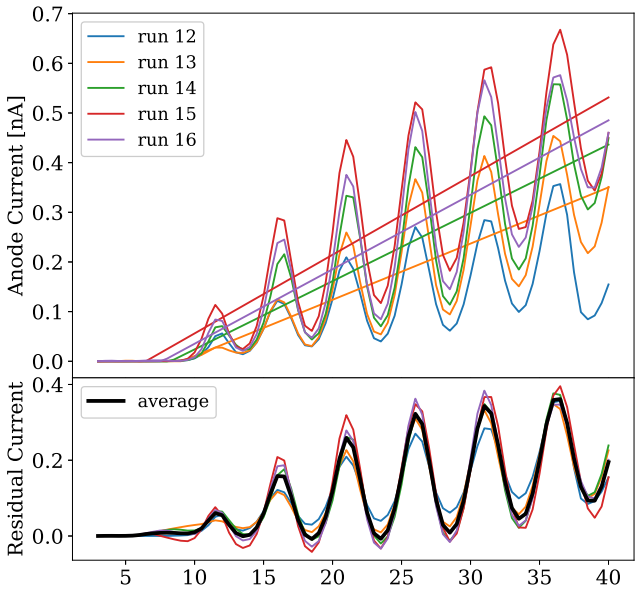

FIG. 5: (Top) Diagnostic runs at $T = 155$ °C with the best-fit line to estimate the bias. (Bottom) Bias-subtracted residual curves and, and the overall average residual. There appears to be a bias present in the amplitude as well, but this should not effect the determination of the minima/maxima locations.

anode current and peak locations, we make five additional diagnostic runs. Each run is measured at a constant temperature of $T = 155$ °C and is recorded with a step size of 0.5 V.

IV. LOCATING MAXIMA AND MINIMA

In order to measure the minima (and maxima) spacings, we first have to locate these extrema. We do this by first smoothing our data using a simple moving average by convolution, using a smoothing window of 1 V. We then calculate the discrete second derivative of the smoothed data, which is searched for zero-crossings (inflection points). The zero-crossings define bin edges, and the data-points within each bin are fit using a 2nd degree polynomial, $I = a + bV + cV^2$. The location of the minimum/maximum is determined by setting the derivative of the best-fit polynomial equal to zero, i.e.

$$V = -\frac{\hat{b}}{2\hat{c}} = \begin{cases} \text{minima} & \hat{c} > 0 \\ \text{maxima} & \hat{c} < 0 \end{cases} \tag{3}$$

To estimate an uncertainty in the location of the observed minima and maxima, we perform the procedure for locating maxima and minima just described on the five diagnostic runs at $T = 155$ °C. Each of these diagnostic runs contains a bias in the current due to leftover electrons from the previous run. The bias is removed by subtracting the difference between a linear fit to the data and a linear fit of the first diagnostic run. The resulting bias-subtracted runs are then searched for minima and maxima. Fig 5 shows the bias-subtraction process. We use the standard deviation of the measured locations of each individual maxima and minima across the five temperature runs as our measure of uncertainty in that peak location. We estimate an overall uncertainty our measurement of the peaks by computing a pooled variance across all located peaks. This uncertainty is expected to be limited by our voltage resolution (0.05 - 0.1 V), and indeed the uncertainties in the minima and maxima locations respectively are found to be $\sigma_{\text{min}} = 0.09$ V and $\sigma_{\text{max}} = 0.07$ V.

The minima/maxima we locate in our data are shown in Fig 3. When defining bin edges, the low S/N regions corresponding to $V < 11$ V are discarded. Likewise, for the run at $T = 135$ °C, we do not search for bins for any voltage > 36 V. The 39 - 40 V region of each run contains artifacts resulting from the smoothing procedure, and is similarly not included in the analysis.

V. DETERMINATION OF THE LOWEST EXCITATION ENERGY OF HG ATOMS

Following the procedure outlined in [3], we plot the spacing between each successive minima as a function of the minima order n. We fit these to Eq 1 and extrapolate the value corresponding to $n = 0.5$ (Fig 6). This value, $\Delta V(0.5)$, corresponds to the smallest energy of an electron required to initiate inelastic collisions with a Hg atom, and is equal to the energy in units of electron-volts (eV). Calculating this quantity for each temperature run,

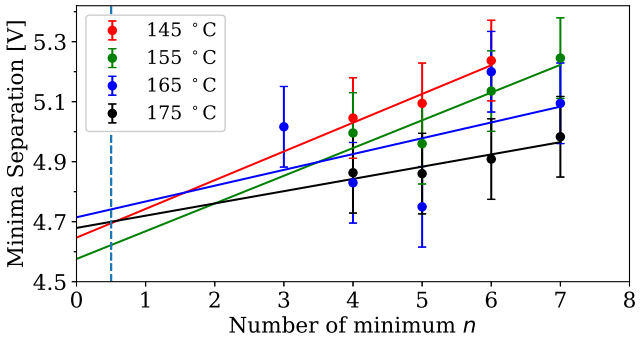

FIG. 6: Minima spacings as a function of the minima order n, and the best fit for each temperature to Eq 1. The dashed vertical line corresponds to $n = 0.5$.

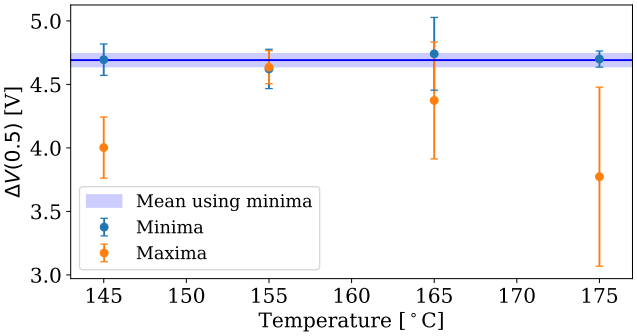

FIG. 7: Measured voltage corresponding to the lowest excitation energy of Hg as a function of temperature. Results using the minima spacings and their mean value with a 68% central interval are shown, as well as results using the maxima spacings.

we determine the mean value to be 4.69±0.05 V, in agreement with the 4.67 eV 6^3P_0 state of Hg.

Several cuts were made on which minima were included in this analysis. The run at $T = 135$ °C was excluded entirely as it has too few peaks to determine a reliable trend in the spacings. The first detected minima was not considered if it is either not significantly pronounced enough or is too "messy." Only the run at $T = 165$ °C satisfies this (the $T = 155$ °C run is is well-pronounced, but odd modulations occurring in the first maximum disrupt the fit). Likewise, the final minima for $T = 145$ °C was not used, as it occurs just before the minima wash away. All maxima are included in the analysis.

We evaluate $\Delta V(0.5)$ using both minima and maxima spacings and plot the result as a function of the tube temperature, as shown in Fig 7. The minima-derived values are consistent with temperature, and the maxima-derived values vary with temperature. This is consistent with the results presented in [3] and is explained the energy of the colliding electrons, which depend partly on the temperature, being at their highest at the observed maxima, and at their most probable value at the observed minima.

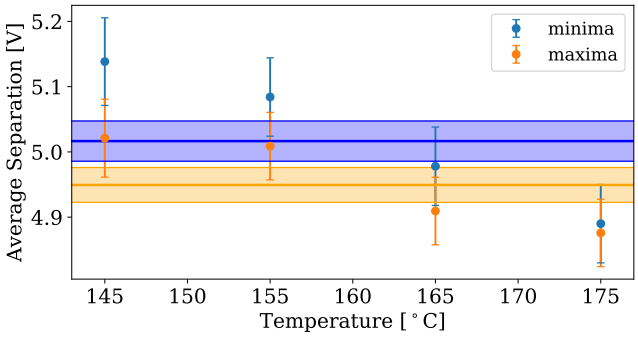

FIG. 8: Average separation of minima (•) and maxima (•) as a function of tube temperature. The overall averages and their 1σ uncertainty for the minima and maxima are given by the filled bands.

Hence the value extrapolated from minima spacings will be consistent, while the value extrapolated from maxima will generally decrease with temperature as the mean free path shortens.

Also note that the slope of the fits decreases with increased temperature. This is reflected by the dependence of the slope on the mean free path of the electrons.

VI. DEPENDENCE OF THE SPACINGS ON THE TUBE TEMPERATURE

In this section we consider the effect of the tube temperature on the average spacing between minima and maxima. Plotting the average spacing as a function of the tube temperature yields the expected decrease, as shown in Fig 8. This is reflected in the example Franck-Hertz curves shown in Fig 4 as well.

Higher temperatures significantly increase the number density of the Hg vapor, which we can assume behaves like an ideal gas. This increased density significantly decreases the electron mean free path, resulting in less "extra energy" to be gained by the electrons once reaching the lowest excitation energy. This in turn causes the spacings of the minima, which depend on this extra energy by Eq 1, to exhibit an overall decrease with temperature. This is further reflected by the slope of the fits in Fig 6 decreasing with temperature, which corresponds to the decrease in the electron mean free path.

The overall average across all temperatures of the average spacing is also shown in Fig 8 by the colored bands. These values can be seen to not agree with the 4.89 V spacing predicted by the lab manual [2] and as quoted by Franck and Hertz in their original experiment [1]. This is not entirely unexpected, as the average spacing between peaks depends on the Hg temperature, pressure, and on tube parameters, and is not exclusively determined by the 4.89 eV transition. In fact, deviations from 4.89 V by a few tenths of a volt is expected and normal [5].

This discrepancy is explained by the model described

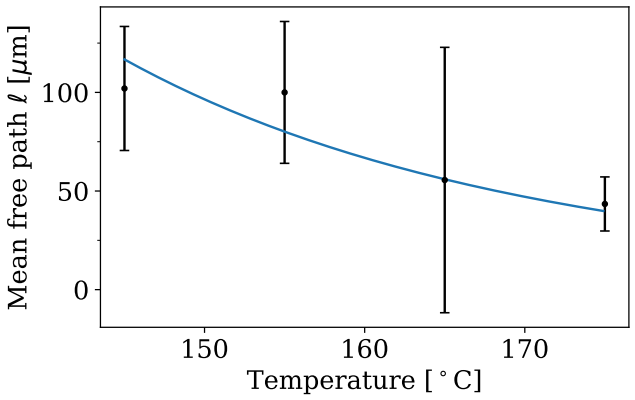

FIG. 9: Mean free path of electrons as a function of the tube temperature.

earlier and proposed in [3]. Electrons reaching 4.67 eV during their journey through the tube will on average travel one mean free path before colliding with a Hg atom. Some of these electrons may gain sufficient energy to excite the 4.89 V transition, and still others may gain enough to excite the 5.46 eV transition (6^3P_2 state). The probability of exciting these higher lines is greater when the mean free path is greater, which occurs at higher temperatures. This further supports the decrease in the average spacing with temperature. Therefore, the average spacing of the peaks in a typical Franck-Hertz curve is a complicated combination of these three excitations, and thus the spacing of peaks depends on which transitions are more dominant than others. Since the mean free path (or equivalently the collisional cross section) is dependent on the tube temperature and pressure, different tube designs will also yield varying results.

VII. DETERMINATION OF THE MEAN FREE PATH OF ELECTRONS IN HG VAPOR AND THE COLLISIONAL CROSS SECTION OF HG ATOMS

Eq 1 allows the determination of the mean free path of electrons in the Hg vapor. Eq 2 gives the expression for the mean free path in terms of the Hg gas parameters, the only unmeasurable of which is the pressure p. Luckily, the pressure of a Hg vapor in the 300 K to 500 K regime is well-approximated by [3, 6][2]

$$p = 8.7 \times 10^{9-3110/T}, \tag{4}$$

where T is given in Kelvin. From our fits shown in Fig 6, the mean free path may be calculated using the cathode-grid distance $L = 1$ cm for our setup. We plot these values vs. temperature in Fig 9 and fit them to Eq 2 using the form of the Hg pressure given above in Eq 4. From this we calculate the cross section for collisions between electrons and mercury atoms to be $(1.1 \pm 0.1) \times 10^{-19}$ m^2. We also calculate the radius of the Hg atom to be 1.87 ± 0.27 Å, in agreement with currently accepted values [2].

VIII. SUMMARY

We conduct the traditional Frank-Hertz experiment for four tube temperatures and determine the lowest excitation energy for Hg atoms to be 4.69 ± 0.05 eV. We explored the dependence of the Franck-Hertz curve on the tube temperature and observed a decrease in average peak spacing with temperature. This phenomena is explained in terms of the mean free path of the electrons, the temperature and pressure of the Hg vapor, and the properties of the tube itself. Finally, we derived the collisional cross section of the Hg atoms and determined the radius of the Hg atom to be 1.87 ± 0.27 Å. This paper is submitted in partial fulfillment of the requirements of PHY 243W: Advanced Experimental Techniques at the University of Rochester.

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[5] G. F. Hanne, American Journal of Physics **56**, 696 (1988).
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[2] The author spent an amount of time longer than he would care to admit wondering if Eq 4 was incorrectly stated in [3]. Only after hunting down [6] in Carlson Library did he realize he had been calculating the pressure using degrees Celsius.