# Black Hole Mergers and Gravitational Wave Astronomy

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# **1** INTRODUCTION

In the year 1915, Albert Einstein published his revolutionary Theory of General Relativity, which generalized his Theory of Special Relativity and Sir Isaac Newton's Laws of Gravitation in describing gravity as a fundamental property of the geometry of spacetime. Einstein's theory argued that matter and energy caused the fabric of spacetime to warp, or curve, by an amount determined by the mass of the matter, and the motion of objects through space was determined by the consequent warping of the spacetime such an object travels in. He formulated this concept into a series of mathematical relations, called the Einstein Field Equations. These relations are the most complete description of gravitation that currently exist, and they are remarkably accurate. Einstein first tested his laws of gravity by calculating the precession of the orbit of the planet Mercury. The precession was not predicted by Newton's laws of gravity, the popular theory at the time, and the cause of the precession was a mystery that had puzzled astronomers for decades. His calculations matched the observed precession precisely [1]. Within a year of the publishing of the Theory of General Relativity, physicist Karl Schwarzschild formulated a solution to the field equations for the gravitational field surrounding a spherical mass, the Schwarzschild Metric [2]:

$$ds^{2} = \left(1 - \frac{\alpha}{R}\right) dt^{2} - \frac{dR^{2}}{1 - \frac{\alpha}{R}} - R^{2} \left(d\theta^{2} + \sin^{2}\theta d\phi^{2}\right), R = \left(r^{3} + \alpha^{3}\right)^{1/3}$$
(1.1)

The Schwarzschild Metric solution for r = 0 results in a gravitational singularity, more commonly referred to as a black hole. Black holes are the ultimate case in Einstein's theory of gravity: they are objects of such incredible density such that the spacetime around them is warped infinitely to a single point, and no objects, not even light, can escape their gravitational grasp. This research report seeks to answer the question of what occurs when two black holes come in contact, and whether the "gravitational radiation" predicted by Einstein in 1916 [3] is produced in this merging of spacetime singularities – and if it is, are astronomers on Earth able to detect it?

# 2 BLACK HOLE MERGERS

Like planets orbiting stars, and stars orbiting other stars, black holes may too orbit other black holes. Such a system could arise from a variety of possibilities. A binary star system in which one star is of sufficient mass at its death that its remnant's mass exceeds the Tolman-Oppenheimer-Volkoff limit [4] and forms a black hole may result in a black hole - star system. If the other star is of sufficient mass as well, the resulting system after the death of both stars could be a binary black hole system. All galaxies are known to contain supermassive black holes at their dynamical centers, and so on these great scales the collision of galaxies could result in the combination of their respective supermassive black holes, creating the ultimate black hole merger.

#### 2.1 MODELING BLACK HOLE MERGERS

As black holes orbit each other in a binary system, we will see later that their motions produce ripples through spacetime in the form of gravitational waves, the same waves that Einstein had predicted from his Theory of Relativity. These waves carry away with them energy from the system, and so the black holes lose orbital energy and thus spiral inward. Eventually the two black holes merge into one. The evolution of a binary black hole system emitting gravitational waves takes place in three stages called inspiral, merger, and ringdown. The stage in which the radiation of gravitational waves causes the binary black hole system to spiral inward is aptly named *inspiral*. The *merger* stage describes the physical merging of the black holes. After merging, the single black hole settles down to a stable form in the *ringdown* stage, during which any remaining distortions in the shape is emitted via even more gravitational waves. [5]

To model what happens when two black holes collide, we can use computer simulations to approximate a binary black hole system. Modeling a system like this using Einstein's equations of relativity can be computationally intensive, and so some constraints are used to simplify calculations. In simulations performed by Frans Pretorius at the California Institute of Technology [6], two non-spinning black holes of equal mass  $M_0$  are given initial separations and orbital velocities such that the inspiral stage lasts only one orbit. Through a numerical code they calculate the evolution of the black hole binary system and find that, after merging, the resultant black hole has a mass of  $1.9M_0$ .





As gravitational waves are emitted from the binary black hole system, the "stretch" in the coordinate distance r from the center of the black hole system is plotted vs various times

throughout the merger event. This is evident as the visible oscillations in Figure 2.1. The first long oscillation in the figure results from the final orbital motion of the two black holes as they merge, and the subsequent waves are the product of the ringdown stage of the final black hole. The total energy *E* emitted in gravitational waves throughout this process can be estimated by integrating the Newman-Penrose scalar  $\Psi_4$  with its complex conjugate  $\bar{\Psi}_4$  over the solid angle of a sphere of constant coordinate radius *R*, and over the total time of the inspiral, merging, and ringdown:

$$\frac{dE}{dt} = \frac{R^2}{4\pi} \int p d\Omega, \text{ where } p = \int_0^t \Psi_4 dt \int_0^t \bar{\Psi}_4 dt$$
(2.1)

For an integration radii of  $25M_0$ , the percent of the initial  $2M_0$  of the binary black hole system radiated in the form of gravitational waves is found to be 4.7%. This is very similar to the difference in mass between initial system mass and final black hole mass, which is  $0.1M_0$  or 5% of the initial energy of the system [6]. From calculations like these we are able to examine what happens if two black holes merge: They combine to form a single, larger black hole, with a mass slightly less than the combined mass of the initial black holes. This difference in mass is the direct result of the emission of gravitational waves throughout the entire black hole merging process, carrying away with them some of the energy of the system.

#### **3** GRAVITATIONAL WAVES

We have discussed what occurs during the merging of two black holes and have quantitatively described the amount of gravitational radiation produced in such an event, but we have not yet discussed what exactly characterizes a gravitational wave. Like any "normal" wave, gravitational waves are also characterized by an amplitude, frequency, wavelength, and a speed of propagation. The speed of propagation for gravitational waves, as Einstein pointed out to Max Born after a conference in Vienna in 1913 [3], is the same as the speed of light. Lets examine gravitational waves as they appear in Einstein's field equations.

#### 3.1 PROPERTIES OF GRAVITATIONAL WAVES

We consider the case of an observer far away from a static matter distribution, and a metric  $g_{\mu\nu}$  to describe the spacetime the observer and mass exist within, the Einstein field equations may be used to determine what happens given a change in our matter distribution. Changing the matter distribution must result in a change in the surrounding gravitational field, and so we will then have a new metric governing our spacetime, say  $\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$ , where  $h_{\mu\nu}$  is the "correction factor" that is the difference in our two metrics. A very useful approximation to make here is to assume that this correction factor is very small, that is  $h_{\mu\nu} \ll 1$ . From this, it can be shown that our correction factor allows for plane wave solutions, like the ones found in electromagnetism, and that these solutions adhere to the three-dimensional wave equation:

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\tilde{h}^{\mu\nu} = 0 \tag{3.1}$$

where here  $\tilde{h}_{\mu\nu}$  is the gravitational field, which is the negative of the trace of  $h_{\mu\nu}$  [7]. The point of this excursion in what seems like hieroglyphics at this level is to show the analogous nature of gravitational waves to electromagnetic waves, and that they share many similar

properties. The difference with gravitational waves is that they are, quite literally, ripples through spacetime itself. Gravitational waves even come in two polarizations! The outcome of a gravitational wave passing through an object would thus be a distortion of the object in a manner of stretching and compressing, in the direction given by the wave's direction and polarization. The two polarizations, plus (+) and cross (×), are demonstrated in Figure 3.1:



**Figure 3.1:** The two polarizations of gravitational waves. On the left, plus polarization is shown to distort objects in spacetime by stretching along one axis and compressing along an orthogonal axis. On the right, cross polarization is shown to have a similar effect, but along an axis rotated 45°. As the wave passes, the distortions oscillate with the wave's frequency. This is shown by the blue and green ellipses, which represent the positions of the particles in the ring at different moments while a gravitational wave passes through the ring. The red circle of particles designates the rest position of the ring of particles [8].

We now have an understanding of gravitational waves as the propagation of ripples in spacetime, and of their analogies to electromagnetic waves. Gravitational waves possess amplitudes describing their strength, frequencies and wavelengths defining the time and distance between successive nodes, a propagation speed equal to that of light, and even polarizations that determine the effects of a wave passing through an object. They are even subject to doppler shifts, like electromagnetic radiation, due to the expansion of space stretching their wavelength, since the waves themselves are embedded in space itself. Since we now have a description of what occurs when gravitational waves pass through an object, perhaps we can devise a method to detect this radiation, and possibly the source of the waves as well. Since we know merging binary black hole systems produce large amounts of this radiation, could we directly observe such a colossal event by means of gravitational wave detection?

#### 3.2 WHY SHOULD WE CARE ABOUT GRAVITATIONAL WAVES?

If we are to invest money in the construction of gravitational wave detectors, we should be able to learn something from them that we could not otherwise learn from electromagnetic radiation, for which we have plenty of functioning detectors. The primary distinguisher between gravitational waves and electromagnetic waves is the *means* by which they are produced. Electromagnetic waves are generally the result of energy transitions within atoms, whereas gravitational waves are the result of the motion of matter through spacetime. Phenomena like binary black hole mergers could be observed directly by measuring the ripples created by the inspiral and ringdown. This event could not be detected via electromagnetic radiation since the black holes do not emit such radiation, and so phenomena like this, or possibly that which occurred before decoupling, may only be observed by means of gravitational waves, as has been the case for each part of the electromagnetic spectrum when first probed.

### **4** GRAVITATIONAL WAVE ASTRONOMY

The first indirect evidence for the existence of gravitational waves was obtained by Joseph Taylor and Russell Hulse, who measured the orbital period of a binary neutron star system to find that the orbital period decayed, and that the decay rate was exactly consistent with the energy-loss resulting from the emission of gravitational radiation. For this indirect detection of gravitational waves by observing the inspiral of two neutron stars, Taylor and Hulse shared the 1993 Nobel Prize in Physics [9]. This discovery drew much attention to the field of gravitational wave astronomy, spurring its growth. Since this discovery, construction began of several observatories designed with the sole purpose of directly detecting gravitational radiation. These detectors are all based on the same principle: Interferometry.

#### 4.1 HOW TO DETECT GRAVITATIONAL WAVES



Figure 4.1: Configuration of the Michelson Interferometer [11]. This type of interferometer was famously used in the Michelson-Morely experiment to show that the supposed "aether" does not exist. The interferometer works by using a 50% reflective, 50% transmissive beam splitter to send a laser down two perpendicular arms. The laser light is reflected by a mirror at the end of each arm, recombined at the beam splitter, and directed into a detector. The detector measures the interference pattern of the recombined laser beam, which would arise if the beams were out of phase due to a difference in the lengths of the arms. This design is the foundation of gravitational wave detectors.

In 1962, M.E. Gertsenshtein and V.I. Pustovoit first described the technique to be used in the construction of a gravitational wave detector [10]. They showed that the effects due to a passing gravitational wave will cause the arm of a Michelson interferometer (see Figure 4.1 above) to change optical lengths, which will produce a detectible displacement in the interference pattern. This would allow interferometers, theoretically, to be capable of detecting even very weak gravitational waves. The energy flux of gravitational waves, like electromagnetic waves, decreases as  $1/r^2$ , and has the form (along the z axis)

$$t_{0z}^{GW} = -\frac{1}{32\pi} \frac{c^3}{G} \omega^2 \left( h_+^2 + h_\times^2 \right)$$
(4.1)

where  $h_+$  and  $h_\times$  are the amplitudes for each polarization [7]. From this, one can determine the energy flux of gravitational waves at Earth. For example, if gravitational waves with energy  $E_{GW}$  on the order of the energy equivalent of  $10^{-4}M_{\odot}$  radiates at a frequency f on the order of 1kHz, for a total signal duration  $\tau$  on the order of 1ms, then the measured amplitude h a distance r away would be

$$h \approx 10^{-22} \left(\frac{E_{GW}}{10^{-4} M_{\odot}}\right)^{1/2} \left(\frac{f}{1 \text{ kHz}}\right)^{-1} \left(\frac{\tau}{1 \text{ ms}}\right)^{-1/2} \left(\frac{r}{15 \text{ Mpc}}\right)^{-1}$$
(4.2)

and so the amplitude, like that of electromagnetic radiation, goes as 1/r as well. This would then give, for a detector arm of length l = 4km, a change in the arm length  $\Delta l = h \cdot l \approx 10^{-22} \cdot 4$ km =  $4 \times 10^{-17}$  cm [7]. If we assume a plus polarized wave traveling along the plane of the interferometer, this will result in one of the arms stretching by this amount and the other compressing by the same amount. This change in length, though, is on the scale of one ten-thousandth the width of a proton! By allowing the light to reflect multiple times within the arms before recombining and being directed into the detector, the effective length of the arm can be increased, allowing for an improvement of several orders of magnitude, but the change in length to be measured is still extremely small. This illustrates the incredible sensitivities that will be required to construct a working gravitational wave detector.

The need for such precision, though, greatly increases the need to be able to filter out sources of noise, of which there are many. Seismic noise, caused by vibrations in the ground from seismic activity or traffic, for example, creates effects 10<sup>9</sup> times stronger than those from gravitational waves [11]. Other limits to the precision include radiation pressure on the mirrors due to the intensity of the laser beam used, "photon shot noise" due to the flux of photons on the detector producing random fluctuations in the interference pattern which reduces the sensitivity of the detector, and, at a fundamental level, the Heisenberg Uncertainty Principle itself [7]. The limit in precision of the instrumentation due to all of these sources, though, can be quantified and addressed, given the specifications of the detector to be built. Now that we know what to look out for, lets start building a gravitational wave observatory.

#### 4.2 FROM THEORY TO HARDWARE

As we have seen, measuring gravitational waves will require extremely precise equipment, the sensitivity of which is limited by various sources of noise. To help minimize disturbances, the arms of the interferometer are evacuated to a vacuum, so the laser is allowed to operate unimpeded. To reduce seismic noise, as shown in Figure 4.2, the mirrors in the interferometer arms are suspended from a quadruple pendulum system. This configuration is capable of reducing seismic noise by 10 orders of magnitude, which is more than is needed to be below the signal strength of gravitational waves!





To reduce the effects of "photon shot noise," high powered lasers are ideal, and photomultiplier techniques can be utilized in the interferometer arms to further increase the strength of the laser. This, coupled with forcing the beam to reflect several times in the arm before recombining, can effectively increase the interferometer arm length while at the same time increasing the intensity of the beam. To achieve an optimal signal, the light should be reflected within the arm for a duration on the order of the signal length, which discussed previously is on the order of 1ms. Thus for an interferometer arm of length 3km, this equates to 50 reflections of the beam, meaning the power of the laser required will be on the order of  $10^3$ W [11]. There are a few different ways to achieve this effect, as demonstrated in Figure 4.3.



**Figure 4.3:** On the left, the path of the beam is diverted to acquire the desired number of reflections, before passing back through the transmissive part of the mirror. On the right, a power recycling mirror is placed in front of the laser which allows the initial beam to pass, but "recycles" the laser by reflecting the beam transmitted by the beam splitter back into the two arms. A signal recycler could also be placed in front of the detector, similar to the power recycler in front of the laser, to limit the bandwidth of the light entering the detector, enhancing its sensitivity [11].

Given a change in length  $\Delta l$  of an interferometer arm, the difference in phase of the two beams is given by  $\Delta \phi = 2b\Delta l/\tilde{\lambda}$ , where  $\tilde{\lambda}$  is the reduced wavelength of the two beams and *b* is the number of reflections each beam makes in the arm before recombining. Typical phase differences should then be on the order of  $10^{-9}$  radians [7]. The initial laser is modulated so that when the beams recombine at the beam splitter, they completely destructively interfere, and thus no light at all reaches the detector. When a gravitational wave causes the length of the arms to change, the phase of the beams are changed, and so the light will no longer destructively interfere. In this case, light with frequency equal to the sum of the frequencies of the laser light and the gravitational wave will pass into the detector [11]. If this light is detected, since the frequency of the laser light is known, it can then be determined what the frequency of the gravitational wave is. The gravitational wave frequencies tend to be on the order of  $10^{14}$ Hz, though, so this is a delicate process.

The final step is to construct a network of multiple detectors at different locations around the world. This way, when a signal is detected in one location, it can be confirmed as an actual gravitational wave event and not an artifact of background noise if an identical event is detected at another location, and corresponds to the appropriate light-travel-time between the two observatories. This also allows for the polarization of the gravitational wave to be determined, given the geometry of the locations of the two detectors. The signals can then be triangulated to determine the position in the sky of the source, and identify the emitter of the gravitational radiation.

#### 4.3 LIGO

The Laser Interferometer Gravitational-Wave Observatory, or LIGO, is a pair of 4km arm length Michelson interferometers designed for the sole purpose of detecting gravitational waves. LIGO is comprised of LIGO Hanford Observatory 4k (LHO 4k), or H1, located in Hanford, Washington, and LIGO Livingston Observatory 4k (LLO 4k), or L1, located in Livingston, Louisiana. The project is funded by the NSF and is organized by the LIGO Scientific Collaboration, which includes over 1000 scientists around the world. The twin LIGO detectors began operations in 2002, and through 2010 conducted joint operations with several other gravitational wave observatories around the world, including TAMA 300 (Japan), GEO 600 (Germany), and Virgo (Italy) [13].



**Figure 4.4:** A schematic of the Advanced LIGO detector. The detector is a Michelson interferometer with 4km length arms which utilizes power recycling, signal recycling, and the suspension of the mirrors on a test mass. (a) shows the locations of H1 and L1, and (b) shows the signal noise at different frequencies. Above 150Hz the noise is dominated by photon shot noise, and at lower frequencies a combination of other noise sources [12].

Throughout this period LIGO completed 5 science runs which established upper limits on gravitational wave sources and constrained noise sources, but no direct signals were measured. In 2007 the two LIGO detectors received several small upgrades to the laser system to double their sensitivity, and "Enhanced LIGO" began operations. In 2010 the two detectors were shut down completely to allow for a large scale overhaul, and upgrades to the full interferometers were completed in 2015. These upgrades increased the sensitivity of the detectors by an order of magnitude, and in September of 2015 "Advanced LIGO" officially began its science runs [11]. Around the same time, on September 14, 2015, the twin LIGO detectors measured a signal that would have made Einstein smile.

#### 4.3.1 GW150914

On September 14, 2015 at 09:50:45 UTC, within three minutes of data acquisition on a new search, L1 and H1 each measured the strongest signal LIGO had yet to detect: GW150914. The delay in measurements at each location were 10ms apart, precisely the light-travel-time between the two observatories [12]. The signal is shown in Figure 4.5.



As one can observe by re-examining Figure 2.1, GW150914 shows great resemblance to the type of signal created by the inspiral and merging of two black holes. Indeed, this is what LIGO had just seen. From the strength of the waves, a total mass of the two black holes can be determined, which is  $\approx 70 M_{\odot}$ . This then gives a lower limit to the combined Schwarzschild radius of the two objects,  $\approx 210$ km. For two objects to orbit with a frequency of 75Hz, which is half the gravitational wave frequency (the wave frequency is "doubled" due to the stretching/compressing of the two arms) measured just before merging, the semimajor axis would only be  $\approx 350$ km [12]. Only black holes can possess such a mass in such a small region. To further confirm this, the signal can be compared to numerical simulations, like those examined in Section 2, of various binary black hole mergers. The results of these reconstructions are show in Figure 4.6.





Both L1 and H1 have an array of various detectors on site, including seismometers, magnetometers, weather sensors, and a cosmic-ray detector, so that measurements of external noise sources can be measured and subtracted from the signal. The false alarm rate of measuring a signal like GW150914 was 1 in 22,500 years, corresponding to a confidence of  $4.6\sigma$ . [12]. The signal was real. For the first time, gravitational waves had been directly detected, and the first binary black hole merger had been observed through ripples in spacetime itself.

#### 4.3.2 THE BLACK HOLES

From examining the GW150915 signal and analyzing reconstructions using Einstein's Theory of General Relativity, the properties of the binary black hole system LIGO observed merging can be determined. The two black holes had masses of  $36^{+5}_{-4}M_{\odot}$  and  $29^{+4}_{-4}M_{\odot}$ , which after merging produced a final black hole of mass  $62^{+4}_{-4}M_{\odot}$ . The mass-equivalent of the energy released in gravitational waves throughout the inspiral and merger was  $3^{+5}_{-5}M_{\odot}$ , roughly 5% of the total mass of the system, as we had predicted should be the case back in Section 2. The luminosity of gravitational wave emission peaked at  $3.6^{+0.5}_{-0.4} \times 10^{54}$  erg/s, on the order of the combined luminosity of all of the stars in the Universe! The gravitational waves were also measured to have a redshift *z* of  $0.09^{+0.03}_{-0.04}$ , which corresponds to a distance of  $410^{+160}_{-180}$ Mpc [12]. This historic first detection of gravitational waves, and direct observation of a stellar-mass binary black hole system, promises to not be the last. LIGO has opened an entire new branch of astronomy that can be used to probe the darkest corners of the Universe. Colossal events like the merging of black holes over a billion light years away can now be directly observed, ironically, through the extreme precision of detecting a minuscule change in length of an interferometer arm here on Earth.

## **5** The Future of Gravitational Wave Astronomy

As evidenced by the success and growth of the first and second generation LIGO and Advanced LIGO experiments, as well as Virgo, GEO 600, and TAMA 300, gravitational wave astronomy is feasible, exciting, and just getting started. While there are plans to continue upgrading the sensitivities of the current second generation detectors, they will still be unable to detect the faintest of gravitational wave sources. There are, however, plans for a new, much higher sensitivity third generation of detectors.

The Einstein Telescope is a European-proposed third generation gravitational wave observatory. Current design plans include three underground interferometer arms, each 10km in length, which will form an equilateral triangle. This would allow for three detectors to be nested within the triangle configuration, with each detector corresponding to the two adjacent arms. This will allow for much more information to be gathered from a single gravitational wave pass, and being underground will greatly help limit seismic noise. Other advancements into laser and mirror suspension systems are also being considered. The Einstein Telescope is expected to reach sensitivities 100 times greater than those currently being achieved in second generation detectors like Advanced LIGO [11].

There are also plans for space-based gravitational wave observatories. The Evolved Laser Interferometer Space Antenna (eLISA), proposed to launch in the 2020s, is an European Space Agency (ESA) funded project aiming to utilize the "quietness" of space to eliminate the many Earth-inherent noise sources. Like the Einstein Telescope, eLISA would include three spacecraft that form an equilateral triangle, each serving as the vertex of a two-armed Michelson interferometer. eLISA would be placed in a heliocentric orbit at a radius of 1AU and would trail the Earth by 20°. Of course, space-based detectors encounter a host of unique challenges that will need to be overcome. A test mission, LISA Pathfinder, is currently operating in space to test many of the systems and issues that will faced by eLISA during its mission [11].

# 6 CONCLUSION

Ever since Einstein first postulated the existence of gravitational waves from his Theory of General Relativity, they have been of great interest to the scientific community. One of many ways that this radiation can be produced is from two black holes orbiting each other. Since the waves carry away energy, they can steal orbital energy from the black holes and cause them to inspiral and merge. This creates a very characteristic pattern of gravitational waves, the exact form of which can be predicted through numerical calculations using Einstein's equations of relativity. In recent years, detectors have been constructed to try and measure gravitational waves by utilizing interferometry techniques to measure the effects of the distortions caused by gravitational waves passing through the Earth. In 2015, the twin LIGO detectors in the United States were able to directly detect gravitational waves produced from the merging of two stellar-mass black holes over a billion light years away. The field of Gravitational Wave Astronomy has been born, and promises many more exciting discoveries to come.

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