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ON MAGNETIC FIELDS, HEATING AND THERMAL CONDUCTION IN HALOS, AND THE SUPPRESSION OF COOLING FLOWS

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ABSTRACT

We discuss the physics of thermal heat transport in halos of giant elliptical galaxies and clusters of galaxies and the consequences for cooling flows in such halos. We demonstrate that—contrary to widespread belief—"tangled" magnetic fields do not reduce the effect of thermal conduction by an amount sufficient to allow one to ignore conduction in cooling flows. Thus, when thermal conduction is included, one can explain the observed density and temperature profiles with the rate of mass inflow reduced from the values commonly quoted in the literature by about a factor of 3–10 for M87. This effect, possibly in conjunction with very modest amounts of heating (possibly as a result of accretion onto, for example, a central black hole), can regulate the cooling flow in a thermally stable manner at mass accretion rates of less than 0.1 $M_\odot$ yr^{-1}.

Subject headings: galaxies: individual (M87) — galaxies: internal motions — galaxies: structure — hydromagnetics — plasmas — X-rays: sources

I. INTRODUCTION

High–spectral resolution X-ray observations with the *Einstein Observatory* of clusters of galaxies have revealed the presence of substantial amounts of relatively warm (on the order of 10^7 K or less) gas in the central regions of these clusters, centered about stationary supergiant elliptical galaxies (Mushotzky *et al.* 1981; Canizares *et al.* 1982). These observations, when taken together with surface brightness profiles that imply cooling times much shorter than the assumed cluster lifetime (e.g., Stewart *et al.* 1984), demonstrate conclusively that radiative cooling is an important energy loss for this gas. Earlier, pre-*Einstein* observations indicated that radiative cooling might be important in some clusters, and Silk (1976) had suggested that cooling matter might accrete onto a stationary central galaxy. This qualitative suggestion was followed by quantitative analyses by Cowie and Binney (1977), Fabian and Nulsen (1977), and Mathews and Bregman (1978), which were used by Gorenstein *et al.* (1977*a*, *b*) to interpret their X-ray data on the Virgo and Perseus clusters in terms of a radiative accretion model. With the *Einstein Observatory*, it became possible to make much more accurate temperature and surface brightness profiles (see Fabricant and Gorenstein 1983), with which the effects of radiative cooling can be studied in more detail. The result is that a number of authors (Canizares *et al.* 1982; Mushotzky *et al.* 1981; Fabian *et al.* 1981; Fabian, Nulsen, and Canizares 1984; Jones and Forman 1984) have interpreted their data to mean that radiative cooling is driving mass accretion onto the stationary supergiant galaxy at a rate given by

$$\dot{m}_0 = 4\pi r^2 n^2 \Lambda(T) \Bigg/ \left\{ \frac{d[5kT/2\mu m_{\rm H} + \phi(r)]}{dr} \right\}, \qquad (1.1)$$

where r is the radius, n is the electron density, $\Lambda(T)$ is the plasma emissivity at temperature T, k is Boltzmann's constant, μ is the mean molecular weight (which is set equal to ≈ 0.6), and $\phi(r)$ is the gravitational potential. The derived mass accretion rates range from 3 $M_\odot$ yr^{-1} for M87 (Canizares *et al.* 1982) to 300 $M_\odot$ yr^{-1} for NGC 1275 (Mushotzky *et al.* 1981; Fabian *et al.* 1981).

There are, however, several peculiarities about this model. First of all, the accretion rate is rather large; it implies that stationary supergiant galaxies have accreted many billions of solar masses of gas. What has become of that gas? Second, if thermal conduction is consistently taken into account, the radiatively driven accretion flows do not produce temperature and density profiles that are consistent with the observations, at least not in the best studied case, M87. Third, where detailed observations are available, equation (1.1) does not give a constant accretion rate, but rather one in which $\dot{m}$ decreases inward, contrary to the assumptions made in deriving equation (1.1). These various problems have given rise to a host of ingenious models which resolve the problems by adopting a variety of *ad hoc* assumptions, such as the creation of many very low mass stars and the suppression of thermal conduction in the halo. The aim of our paper is to show that the problems with the cooling flow models can be resolved in a straightforward manner by suppressing, not thermal conduction, but the accretion rates, to very low values. This reduction can be effected by a combination of thermal conduction and internal heating (see Tucker and Rosner 1983; Bertschinger and Meiksin 1986; Friaca 1986; Silk *et al.* 1986; Meiksin 1987; Boehringer and Morfill 1987).

We discuss in more detail in § II the difficulties of cooling flow models and how these difficulties can be resolved with thermal conduction and internal heating. In §§ III–V we consider objections that have been raised to this resolution. We show that a stable form of cosmic-ray heating is possible (§ III), that the possible reduction of thermal conduction by tangled magnetic fields has been greatly overestimated (§ IV), and that fine tuning of heating and radiative losses can be achieved in a self-regulating manner (§ V); our conclusions are summarized in § VI.

II. ESTIMATES OF MASS ACCRETION RATES IN COOLING FLOWS

We contend that the mass accretion rates in cooling flows have been greatly overestimated. As several authors (Fabian, Nulsen, and Canizares 1982; Canizares *et al.* 1982; Lea, Mushotzky, and Holt 1982; Sarazin and O'Connell 1983) have noted, the accretion rates derived from equation (1.1) are surprisingly large. Can one confirm these accretion rates by independent means? It has been argued that these large accretion rates are observationally confirmed by optical data. However, the optical data in fact do not constrain the mass accretion rate any more than do the X-ray data. In most discussions of this question (e.g., Cowie *et al.* 1983; Fabian, Nulsen, and Canizares 1984), the Hα emissivity is related to the mass accretion rate by assuming that each hydrogen atom produces an Hα photon as it cools. This is equivalent to the assumption that the cooling time is much shorter than the ionization time and does not yield the correct Hα emissivity; hence shock waves and heating from a central source are invoked to yield approximately 5 Hα photons per cooling atom (the factor 5 is roughly the ratio of the cooling time to the recombination time). If heating by conduction and other sources is effective, the cooling time could be orders of magnitude larger, and the same Hα emissivity could then be produced by an accretion rate that is orders of magnitude smaller.

But let us for the moment assume that the large derived accretion rates are correct. Then, if radiative accretion has been occurring at essentially these presently observed rates for the entire lifetime of the clusters, then the central galaxies must have accreted between 3×10^{10} and 3×10^{12} $M_\odot$ of gas. What has become of this gas? As discussed in the papers just cited, it is not in the form of cooler gas because it would have shown up in optical observations of Hα if the gas were in the neighborhood of 10,000 K or in radio observations of H I at 21 cm if the gas were much cooler. If the gas had remained hot, on the order of several hundred thousand degrees, it would be detected either by soft X-ray observations or through the pressure it exerted on the optical filaments. Nor is it possible to hide a 10^{10} $M_\odot$ black hole in the nucleus of a galaxy. It would be detectable both through its effects on the stellar dynamics of the galaxy and through the energy released by accretion of several $M_\odot$ yr^{-1} of gas into the black hole:

$$L \cong 6 \times 10^{45} \dot{m} \text{ ergs s}^{-1} , \tag{2.1}$$

where $\dot{m}$ is the accretion rate in $M_\odot$ yr^{-1}, and the efficiency of rest mass energy conversion in the accretion process has been taken to be equal to 0.1.

Sarazin and O'Connell (1983) have considered in some detail the possibility that the accreted matter forms low-mass stars (see also Fabian, Nulsen, and Canizares 1982, and Cowie and Binney 1977). The formation of high-mass stars is ruled out by lack of evidence of a large population of young, blue stars in these galaxies. However, by making appropriate assumptions about the initial mass function, Sarazin and O'Connell (1983) were able to construct accretion-driven star formation populations with spectra consistent with the observations. For example, the upper limit of the mass spectrum has to be on the order of 1 $M_\odot$ for M87, and on the order of 2.8 $M_\odot$ for NGC 1275. They argue that such mass spectra are to be expected because the Jeans mass varies as $T^2 p^{-1/2}$ for a protostellar cloud of temperature T and pressure p, and the pressures expected from accretion flows are several orders of magnitude higher than in the local interstellar medium. However, the pressures in accretion flows are comparable to those in dense molecular clouds, which are thought to be some of the most prolific sites of star formation in our galaxy, and stars of a solar mass or higher appear to have no difficulty forming there (Miller and Scalo 1979). The possibility of hiding large amounts of cooling matter from accretion flows in low-mass stars therefore remains speculative, with no empirical basis.

Another possibility mentioned by Lea, Mushotzky, and Holt (1982) and discussed in some detail by Tucker and Rosner (1983, hereinafter TR) is that the mass inflow rate has been overestimated by a large factor. This would happen, as discussed below, if either thermal conduction is ignored, or if an additional substantial heat source were present, or both. In the presence of heating h by thermal conduction and by a central source, equation (1.1) becomes

$$\dot{m} = 4\pi r^2 (n^2 \Lambda - h) \Big/ \left\{ \frac{d}{dr} \left[\frac{5kT}{2\mu m_\mathrm{H}} - \phi(r) \right] \right\}$$
$$= \dot{m}_0 \left(1 - \frac{h}{n^2 \Lambda} \right) . \tag{2.2}$$

Equation (2.2) shows that the accretion rate can be arbitrarily small for an arbitrarily fine balance between net heating and radiative losses. Introduction of a heating term into the energy balance equation can, in principle, provide a simple solution to the problem of hiding excessive amounts of accreted mass: very little mass is in fact accreted, so there is little or no mass to hide.

The actual accretion rate depends on how delicately the radiative and heating terms are balanced: it is impossible to compute this accurately, but an order of magnitude estimate can be obtained from other considerations. Thermal conduction can effectively balance radiative losses outside about 30 kpc. Inside 30 kpc, the difference between radiative losses and thermal conduction rises until it reaches a maximum of about 3×10^{42} ergs s^{-1} (Tucker and Rosner 1983; Stewart *et al.* 1984). This difference could be supplied by cosmic-ray heating in the central 10–20 kpc of the halo. For example, we can estimate from equation (2.1) that a mass accretion rate of about 0.01 $M_\odot$ yr^{-1} into a central black hole would release $\approx 6 \times 10^{43}$ ergs s^{-1}. This would be enough to explain the energy output of the nucleus and the associated jet. If 5% of this energy could be channeled into relativistic electrons in the radio lobes of M87, the combined effects of thermal conduction and collective heating by relativistic electrons could balance radiative losses (similar values for the cosmic-ray energy flux in M87 have been arrived at on the basis of completely independent arguments by Boehringer and Morfill 1987). Schreier, Gorenstein, and Feigelson (1982) have discussed how spherically symmetric accretion, with an efficiency for producing X-rays of $\approx 10^{-4}$ at a rate of 0.01 $M_\odot$ yr^{-1}, would explain the X-ray source associated with the nucleus of M87. An accretion rate of this magnitude would increase the mass of the black hole by 10^8 $M_\odot$ over 10^{10} yr, well within Dressler's (1980) limit of 5×10^8 $M_\odot$ for a massive object.

Stewart *et al.* (1984, hereinafter SCFN) have argued that this solution to the problem of excessive accreted matter is untenable because (*a*) an equilibrium solution with a localized heat source is locally unstable, (*b*) thermal conduction is unimportant because the magnetic field lines are tangled, and (*c*) the solution requires fine tuning of the boundary conditions on temperature and density in the halo to achieve the proper balance between radiative losses (which are efficient at high densities and low temperatures) and thermal conduction (which is most efficient at high temperatures. We now consider each of these objections in turn.

III. THE THERMAL STABILITY OF HALOS

In this section, we focus very briefly on the stability to local perturbations properties of cooling, gravitationally confined atmospheres (of which halos are an example). Mathews and Bregman (1978), Cowie, Fabian, and Nulsen (1980), SCFN, and Nulsen (1986), among others, have discussed approximate thermal stability criteria for halos, based on a simple "bubble" picture of the fluid, while, more recently, White and Sarazin (1987*a*, *b*) and Malagoli, Rosner, and Bodo (1987) have addressed this problem from a more rigorous perspective by linearizing the full set of hydrodynamic equations for a gravitationally stratified fluid, and deriving the appropriate local dispersion relation. The latter authors focused in particular on the nature of the unstable modes in halos, using as a particular example the halo of M87 (with an assumed background mass distribution given by model C of SCFN) in the region 4–100 kpc from the galactic core. Using two models (model I—no thermal conduction, mass inflow rate $\dot{m} = 3(r/10\ \text{kpc})^{2/3}$ $M_\odot\ \text{yr}^{-1}$ (SCFN); model II—100-fold reduction in thermal conduction efficiency, constant $\dot{m} = 0.1\ M_\odot\ \text{yr}^{-1}$), Malagoli, Rosner, and Bodo showed that in both models, local isobaric perturbations are thermally overstable as long as the perturbations were not too flattened in the direction perpendicular to gravity; the latter "pancake-like" modes, whose wave vector $\boldsymbol{k}$ is aligned with the local gradient of the gravitational potential, are instead monotonically unstable, with somewhat larger growth rates than the overstable modes (see White and Sarazin 1986). Such modes, however, lead to density inversions which are themselves Rayleigh-Taylor unstable: That is, it is readily shown that the Rayleigh-Taylor growth rate $\omega_{\text{RT}} \equiv |\boldsymbol{k} \cdot \boldsymbol{g}|^{1/2}$ (where $\boldsymbol{g}$ is the local gravitational acceleration) is larger than the local buoyancy frequency $\omega_{\text{BV}} \equiv [(\gamma - 1)g/c_s^2 k]^{1/2}$, which in turn is substantially larger than the radiative cooling rate for the perturbations in question; hence these "pancake" modes ought to fragment rapidly, leading again to "cigar-like" modes aligned with $\boldsymbol{g}$ (which are overstable). However, Malagoli, Rosner, and Bodo also show that all such cigar-like perturbations whose wavelengths are small enough to be consistent with the local nature of the dispersion relation are damped by thermal conduction for values of the thermal conductivity as small as 0.01 of the Spitzer value. Hence, unless thermal conduction is suppressed by factors substantially greater than 100 from the nominal Spitzer value, local isothermal perturbations will be suppressed. For this reason, it is essential to establish whether thermal conduction is indeed inhibited and, if so, to what extent (see § IV).

So far, we have ignored the possible role which local heating may play in regulating the thermal stability characteristics of the halo gas. In order to keep the discussion simple, let us for the moment ignore the possible role played by thermal heat conduction. In that case, the relevant physical processes are simply those which lead to a net energy input or loss from the halo as a whole: radiation, accretion, and direct heating (such as possibly by thermalization of relativistic particles; see below). We therefore apply the criterion for the stability of such a radiatively cooling atmosphere in the presence of a heat source given by Field (1965) to the halo taken as a whole,

$$\frac{d[\bar{h} - n^2\Lambda(T)]}{dT} < 0\ , \tag{3.1}$$

where $\bar{h}(n, T)$ is a net heating function, Λ is the plasma emissivity per unit emission measure, and the overbar signifies averaging of the local heating rate over times longer than the acoustic crossing times for the halo, but shorter than the typical thermal instability time scales. For constant pressure (isobaric) perturbations, inequality (3.1) becomes

$$2 - \frac{d\log\Lambda}{d\log T} < \left(\frac{\partial\log\bar{h}}{\partial\log n} - \frac{\partial\log\bar{h}}{\partial\log T}\right)\left(\frac{\bar{h}}{n^2\Lambda}\right)\ ; \tag{3.2}$$

if $\Lambda(T) = 10^{-19}T^{-x}$ (where $x \approx \frac{1}{2}$ for $10^6 < T < 10^{7.5}$ K), then the stability criterion (3.2) becomes

$$\frac{\partial\log\bar{h}}{\partial\log n} - \frac{\partial\log\bar{h}}{\partial\log T} > 2.5\left(\frac{n^2\Lambda}{\bar{h}}\right)\ . \tag{3.3}$$

For cooling flows, the mean local heating rate must be less than the mean radiative loss; in the cases we consider, this inequality also holds for the unaveraged quantities. Hence, $n^2\Lambda > \bar{h}$, and the bracketed ratio on the RHS of equation (3.3) is greater than 1. If, contrary to the assumption which led to equation (3.3), heat conduction is important (as it will be at least in the outer parts of the atmosphere), it works to aid stability, as just discussed above (Field 1965).

To summarize, the above inequalities require that, for the gas to be stable, any decrease in temperature due to radiative cooling must be offset by an increase in heating at the lower temperatures; this can only occur if the heating rate is functionally related to ambient conditions in the halo, e.g., $\bar{h} = \bar{h}(n, T)$ such that equation (3.3) holds. The problems are, first, to deduce this functional dependence and, second, to demonstrate that this functional dependence has the proper form. The first problem is a complex one that involves an understanding of, among other things, the processes for generating high-energy particles in the nuclei of galaxies. We do not address that problem here. Rather, we merely suggest that the existence of a heating function that satisfies (3.3) is plausible, contrary to the assertion of SCFN.

As an example of the possible functional form of the heating term, consider heating by cosmic-ray electrons, a mechanism which is likely to be significant only if collective effects are important. The heating rate is then given by

$$h \approx \eta j^2\ , \tag{3.4}$$

where j is the current density of the relativistic electrons, with number density n_r; e.g.,

$$j = n_r ec\ ; \tag{3.5}$$

η is the plasma resistivity,

$$\eta = \frac{4\pi}{\omega_p}\frac{W_{\text{pl}}}{3nkT}\ , \tag{3.6}$$

where ω_p is the nonrelativistic plasma frequency and W_{pl} is the energy density in plasma waves (Scott *et al.* 1980). If we assume that $W_{\text{pl}} \propto n_r\langle\gamma\rangle mc^2$ and that $\langle\gamma\rangle$ is solely a property of the source of the relativistic electrons, then

$$h \propto n_r^3 n^{-1.5} T^{-1} . \quad (3.7)$$

If we further assume that the relativistic electron pressure is proportional to the magnetic pressure and that the relativistic electrons are tied to the lines of magnetic flux, then, as a blob of gas is compressed, magnetic flux conservation implies $n_r \propto n^{4/3}$, so that

$$h \propto n^{2.5} T^{-1} , \quad (3.8)$$

which satisfies the stability criterion (3.3). The underlying physics of this situation can be readily understood. If a cooling blob of gas collapses, then the lines of magnetic flux will be concentrated into a smaller volume, producing a larger heating rate, which offsets the radiative cooling. Note that magnetic mirroring (which could occur if the pitch angles of the relativistic electrons are large) will not change this argument since the electrons will spend most of their time at the mirror points; hence their density will still be increased in the compressed regions.

To conclude, whether a given halo is thermally stable on interesting temporal and spatial scales is largely a function of two processes—thermal conduction and direct plasma heating—about which we have little direct observational information. It therefore behooves us to consider in detail any other information which can be used to constrain these two processes; and in the following, we focus, in particular, on the physics of thermal conduction.

IV. THERMAL CONDUCTION IN THE PRESENCE OF MAGNETIC FIELDS IN HALOS

In this section, we discuss the possible suppression of thermal conduction in galactic and cluster halos. To begin with, we note that the evaluation of thermal conduction in a magnetized fluid is a long-standing problem in both laboratory and astrophysical plasma physics, and we do not aim (or claim) to cover entirely new ground here: In the present context, the basic question is whether there is any process which will reduce the effective thermal conductivity in the halo of a cluster below that given by the Spitzer value, assuming that magnetic fields do thread the halo.

a) Is Thermal Conduction "Classical"?

To answer the general question regarding thermal conduction fully, we must first consider several subsidiary questions. The first is whether the (classical) Spitzer-Braginskii formalism for calculating the heat flux ought to apply. A sufficient condition for the validity of this formalism is found by evaluating the ratio of the thermal mean free path $\lambda_{\rm mfp}$ of electrons to the radial gradient scale length of the halo temperature distribution L_T for typical radial halo temperature distributions (see Fabricant, Lecar, and Gorenstein 1980), which yields

$$\lambda_{\rm mfp}/L_T < 10^{-2} ; \quad (4.1)$$

as shown by, for example, Gray and Kilkenny (1980), this condition is sufficient to be in the classical heat conduction domain. We can demonstrate this point more explicitly by means of the following argument. Consider for simplicity a spherically symmetric atmosphere in which the minimum and maximum temperatures are attained on the boundaries and in which the divergence of the conductive heat flux locally balances radiative losses throughout (e.g., we assume that there are no other sources of energy, such as an inward enthalpy flux); this balance between conduction and radiation maximizes the heat flux (because the heat flux must carry in all of the energy lost to radiation). If the heat flux vanishes at the lower temperature boundary $r = r_0$, we can immediately obtain the maximum heat flux, which is attained at the outer boundary $r = r_1$,

$$F_{\rm max} \equiv F(r = r_1) \approx 8 \times 10^{-7} (r/10^{24}\ {\rm cm})^{-2} \times (L_x/10^{43}\ {\rm ergs\ s^{-1}})\ {\rm ergs\ cm^{-2}\ s^{-1}} ; \quad (4.2)$$

thus

$$\lambda_{\rm mfp}/L_T \approx 10^{-4} (n/10^{-3}\ {\rm cm^{-3}})^{-1} (T/10^7\ {\rm K})^{-3/2} \times (r/10^{24}\ {\rm cm})^{-2} (L_x/10^{43}\ {\rm ergs\ s^{-1}}) (\ln \Lambda/20)^{-1} . \quad (4.3)$$

Hence the condition for the validity of the classical Spitzer-Braginskii linearization of the electron velocity distribution function is satisfied even under rather extreme assumptions about the magnitude of the heat flux; for example, constraint (4.1) is satisfied even better at densities and temperatures more characteristic of the outer reaches of halos such as that of M87 (e.g., $n \approx 10^{-5}$–10^{-4} cm^{-3}, $T \approx 3 \times 10^7$ K). We do caution, however, that the linearization may nevertheless fail when the mean free path of heat-flux–carrying electrons becomes comparable to the local temperature gradient scale length; in this case, one can show that, for the case at hand, the Spitzer conductivity leads to a slight overestimate of the heat flux and its divergence in the outer parts of the halo (see Luciani, Mora, and Pellat 1984).

Equation (4.2) also can be used to demonstrate that the maximum heat flux lies well below any standard threshold for the excitation of plasma waves which might increase the effective electron scattering frequency (and hence reduce the effective parallel thermal conductivity; see Max 1981); hence one cannot appeal to "anomalous" heat conduction in such halos. Furthermore, the time scale on which thermal contact via conduction is established within a halo (subject to the restrictions imposed by magnetic fields, discussed further below) is of the order of the dynamical (sound crossing) time scale and hence less than any of the time scales of concern to us here. To conclude: *in the absence of magnetic fields, thermal conduction is well described by the classical Spitzer-Braginskii formalism throughout most of the halo.*

b) What does "Tangling" of Magnetic Field Lines Do to the Effective Thermal Conductivity?

The second question is whether "tangled" magnetic fields can significantly reduce the *effective* thermal conductivity. Some authors have claimed that this effect can essentially suppress thermal conduction entirely as a significant contributor to the halo's energy budget (Binney and Cowie 1981; Stewart *et al.* 1984; Fabian, Nulsen, and Canizares 1984). In order to assess the validity of this claim, it is helpful actually to calculate explicitly the change in the nature of heat transport when the magnetic field is "tangled."

To begin with, field line "tangling" will only be important to the extent that the magnetic field is strong enough to effectively channel the motion of thermal electrons; the question is then how small the magnetic field strength can be and still inhibit thermal conduction across the local magnetic field direction. The effectiveness of such inhibition is often judged by comparing the typical scale lengths L in a galactic or cluster halo with the gyroradius r_e of a thermal or a heat-flux–carrying electron

$$r_e/L = 2.21 \times 10^{-2} T_e^{1/2} B^{-1}/L$$
$$\approx 7.4 \times 10^{-15} (T/10^8 \text{ K})^{1/2} (B/10^{-6} \text{ G})^{-1} (L/10 \text{ kpc})^{-1} ,$$

which is an extremely small number for plausible values of the halo magnetic field strength. In that case, it is argued, one can adopt the value given by Spitzer (1962) for the perpendicular thermal conductivity κ_{perp}, which is indeed many orders of magnitude smaller than the parallel thermal conductivity κ_{parallel} under such conditions.

This line of reasoning, however, neglects the dynamical effects of thermal heat conduction. In a laboratory plasma, magnetic fields which satisfy the conditions that $r_e/L \ll 1$ are also typically sufficiently strong for the associated Lorentz forces to overwhelm the thermal heat flux force completely; hence, the magnetic field tends to channel the heat flux. This regime also obtains under solar conditions, as well as under conditions typical of the interstellar medium, but may not obtain under the conditions commonly assumed for galactic and cluster halos.

To see this quantitatively, we compare the component of the thermal force perpendicular to the local magnetic field, $F_{T,\text{perp}}[\equiv (3nk_B/2\omega_{ce}\tau_e)\nabla_{\text{perp}} T_e]$, with the opposing Lorentz force generated as field lines are bent, $F_B[\equiv \boldsymbol{B} \cdot \nabla \boldsymbol{B}/4\pi]$, where ω_{ce} is the electron cyclotron frequency, τ_e is the mean electron collision time, and T_e is the electron temperature. The ratio of these two opposing forces is given by

$$R \equiv \frac{F_{T,\text{perp}}}{F_B} \approx 4 \times 10^{-15} \left(\frac{n}{10^{-4} \text{ cm}^{-3}}\right)^2 \left(\frac{T}{10^8 \text{ K}}\right)^{-1/2} \times \left(\frac{B}{10^{-6} \text{ G}}\right)^{-3} \frac{\ln \Lambda}{20} .$$

Evidently, the force due to the component of the thermal gradient perpendicular to the local magnetic field is comparable to, or larger than, the Lorentz force for a critical magnetic field strength

$$B_{\text{crit}} \approx 1.6 \times 10^{-11} \left(\frac{n}{10^{-4} \text{ cm}^{-3}}\right)^{2/3} \times \left(\frac{T}{10^8 \text{ K}}\right)^{-1/6} \left(\frac{\ln \Lambda}{20}\right)^{1/3} \text{ G} ,$$

or smaller under conditions likely to prevail in galactic or cluster halos. The essential question is then whether the magnetic field strength in halos such as that of M87 is smaller or larger than this critical value; what few data exist to answer this question (Kim *et al.* 1986; Feigelson *et al.* 1987) suggest that field strengths lie in the range of $(0.1\text{–}1) \times 10^{-6}$ G and hence well above B_{crit}; the heat flow is therefore most likely channeled by the halo magnetic fields.

If we then assume that the actual field strength exceeds the critical value B_{crit}, we next need to clearly define what we mean by a "tangled" magnetic field. In the problem at hand, the relative ordering of the physically meaningful length scales is

$$\lambda_{ge} < L_B < \lambda_{\text{mfp}} \ll L_T , \tag{4.4}$$

where λ_{ge} is the thermal electron gyroradius and L_B is the typical scale length of magnetic field fluctuations. In this regime, temperature is a locally well-defined thermodynamic quantity, thermal conduction along the magnetic field obeys the classic Spitzer-Braginskii formalism (as we just demonstrated above), and collisions are sufficiently infrequent (and the thermal electron gyroradius sufficiently smaller than the length scale of typical magnetic field fluctuations) that the gyrocenters of electron orbits closely follow magnetic field lines. In these limits, thermal contact is only maintained between points which are connected by magnetic field lines; and for these points, the effect of tangling magnetic field lines is primarily to increase the total path length which heat-flux–carrying electrons must traverse.

The first point is thus to establish the likely field topology within the halo (Fig. 1). For example, can one arrange to have enclosed $\boldsymbol{B}$ field "bubbles" (Fig. 1*a*) whose scale length $L_B \ll \lambda_{\text{mfp}}$? If so, then such bubbles (or clouds) will be relatively thermally insulated. The difficulty with such a field topology lies in its generation. In order to form disconnected magnetic flux regions, it is necessary to invoke reconnection; unfortunately, the associated time scales for classical resistive processes are enormous under the plasma conditions prevailing in halos (and collective scattering processes (such as scattering by electrostatic plasma waves) only operate under extreme current density regimes not likely to prevail in the halo gas). In any case, since we know from the virial theorem that such self-contained field structures are unstable unless externally "tied," it appears far more likely that dissipative processes would actually lead to the merger of such magnetic field "loops" (i.e., coalescence) rather than to their formation. An alternative, more plausible, field topology arises if the halo gas has been ejected from many of the cluster member galaxies, each one contributing a bubble of gas, with entrained magnetic fields "looping back" to the parent galaxy (Fig. 1*b*); this field connectivity would ensure that "bubbles" remain relatively isolated from one another. In this case, gas within a given bubble would be thermally well connected, and hence any given bubble ought to appear rather isothermal throughout most of its volume. However, given that different bubbles must have had rather different histories, and hence have experienced rather different adiabatic cooling via expansion, different initial conditions, etc., it remains to explain how the observed homogeneity in temperature in halos such as that of M87 is obtained without invoking conduction between distinct bubbles. In the absence of such an explanation, it seems to us far more plausible that the entrained fields carried out with the expelled gas from individual cluster galaxies will to some extent merge at the contact surfaces between contiguous bubbles (because magnetic field gradients in these boundary layers, formed as adjacent bubbles are forced together, can become sufficiently large to reduce the reconnection times below the cooling time scale for the halo itself). In that case, the effect of such bubbles on thermal transport will be largely to increase the total path length which heat-flux–carrying electrons must traverse from the outer (hot) to the inner (cool) boundaries of the halo; a similar effect would result if the halo gas were highly turbulent.

Given that the effective heat transport path lengths are increased, there is a straightforward method for evaluating the consequent effect on heat transport; that is, one calculates the effective mean free path for heat-flux–carrying electrons in the presence of magnetic field fluctuations and thus directly evaluates the effective heat conductivity along the mean direction of the heat flux (and magnetic field), e.g., along the radial direction. In this case, the random field fluctuations lead to an increase in effective path length, which is taken into account by using the statistics of the field fluctuations to calculate the

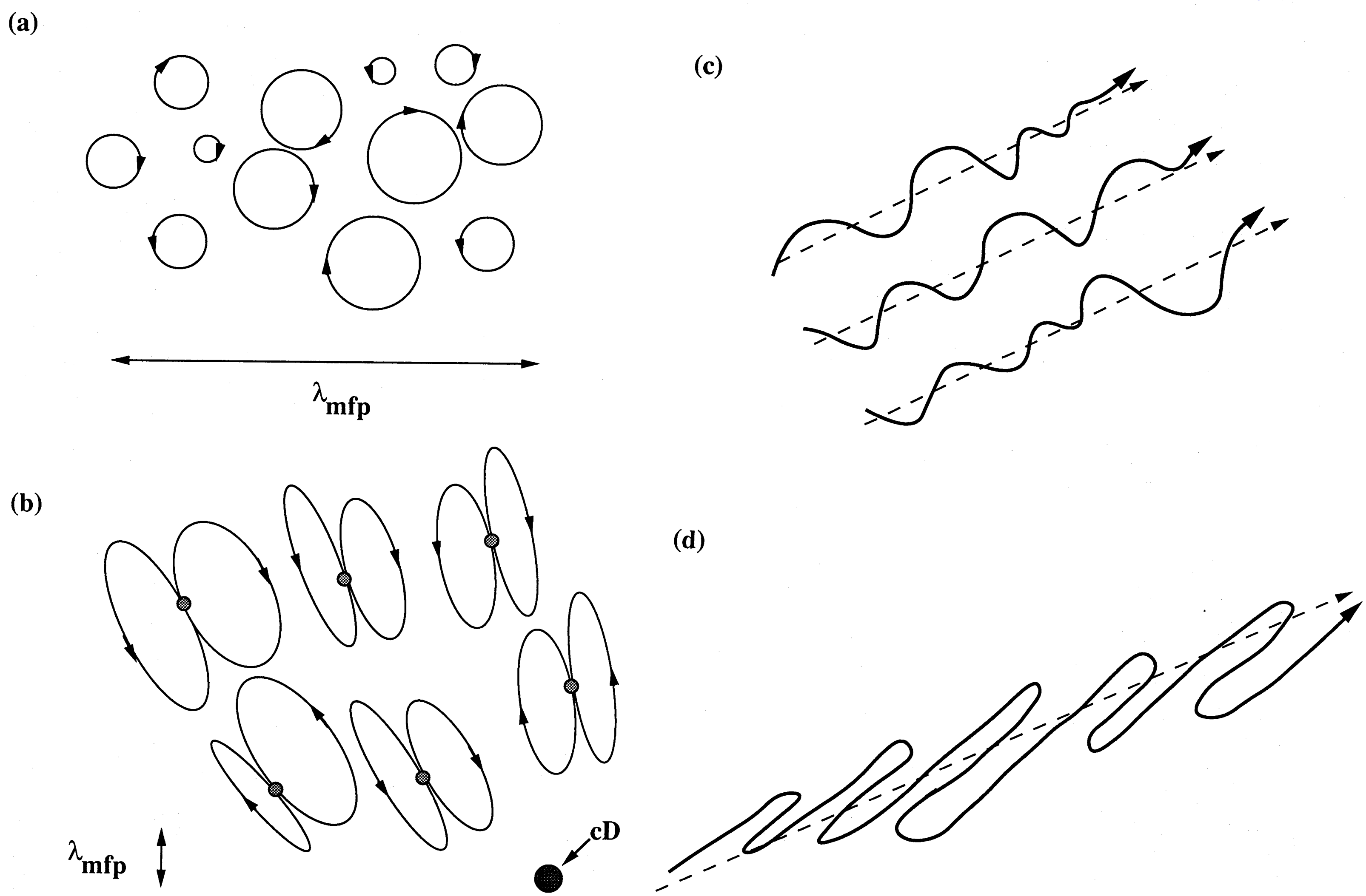

FIG. 1.—Four extreme prototypes of magnetic field tangling: (*a*) magnetic field lines are entirely locally connected within bubbles, which are in consequence thermally insulated from one another; the field scale length is of order or larger than the electron mean free path, so that individual bubbles are largely isothermal; (*b*) field geometry basically as in (*a*), but here the bubbles are the result of entrained magnetic fields carried out by gas flowing from individual cluster member galaxies; note that in cases (*a*) and (*b*), there is no readily defined large-scale magnetic field direction; (*c*) magnetic field lines are globally connected, and local magnetic field deviations are dominantly perpendicular to the mean field direction; (*d*) magnetic field lines are globally connected, and local magnetic fluctuations are aligned with the mean magnetic field direction. In the latter two cases, the direction of the mean magnetic field is roughly indicated by the dash.

effective range, or mean scattering distance, of heat-flux–carrying electrons. This calculation is fundamentally based on studies of random walks of magnetic fields, and the resulting scattering of cosmic rays, by Jokipii and Parker (Jokipii 1966; Jokipii and Parker 1969; Jokipii 1971) and on the explicit evaluation of the consequent heat flux reduction in the solar wind by Hollweg and Jokipii (1972).

In order to derive the magnitude of the change in the local heat flux due to a possible reduction in the effective heat conductivity, we follow the formalism of Hollweg and Jokipii (1972), who first evaluated the effective heat conductivity in the presence of stochastically oriented magnetic fields under (among others) precisely the assumed spatial scale ordering embodied in equation (4.4). In this physical domain, the effective mean free path in the direction defined by the large-scale temperature gradient is

$$\lambda_{\text{eff},\nabla T} = \lambda_{\text{mfp}}\langle \Delta z/\Delta s\rangle = \lambda_{\text{mfp}}\langle \cos\,\delta\theta\rangle\ , \qquad (4.5)$$

where λ_{mfp} is the thermal mean free path, Δz is the projected distance between successive scattering events along the direction of the mean magnetic field $\langle \boldsymbol{B}\rangle$, Δs is the actual distance traveled by an electron along the local magnetic field between successive scattering events, $\delta\theta$ is the local angle between $\boldsymbol{B}$ and $\langle \boldsymbol{B}\rangle$, and the angular brackets denote ensemble averaging (which, under the ergodic hypothesis, is equivalent to averaging along the orbits of test electrons). If (1) the magnetic field fluctuations are dominated by changes in the direction of $\boldsymbol{B}$, rather than in its amplitude, (2) $\cos\,\delta\theta > 0$ (so that magnetic field lines do not loop back on themselves; see below), and (3) we assume for simplicity that the temperature gradient lies in the direction of the mean magnetic field, then the reduction in the thermal conductivity which corresponds to this effective reduction in the scattering mean free path can be shown to be

$$\kappa_{\text{eff},\langle \boldsymbol{B}\rangle} = \kappa_0\langle \cos\,\delta\theta\rangle^2\ ; \qquad (4.6)$$

κ_0 is the classical Spitzer conductivity along the local magnetic field, while $\kappa_{\text{eff},\langle \boldsymbol{B}\rangle}$ is the effective conductivity along the direction of the mean magnetic field (and, by assumption, the temperature gradient). Note that there is an additional factor of $\langle \cos\,\delta\theta\rangle$ from that quoted by Binney and Cowie (1981).

If magnetic field lines do loop back, then equation (4.6) can be trivially modified as follows. In the limit that the gyroradius is much smaller than $L_{\boldsymbol{B}}$, and as long as (4.4) is satisfied, consider the two extreme cases in which we can decompose the particle motion into either a sequence of large-scale "loop-backs," on which are superposed local deviations perpendicular to the mean field direction (Fig. 1*c*), or a sequence of small-scale loop-backs superposed on larger scale deviations perpendicular to $\langle \boldsymbol{B}\rangle$ (Fig. 1*d*). The case we must resolve is the former of the two. Consider then any field line of the form shown in Figure 1*c*. To evaluate equation (4.6) along it, we break the field line up into segments along which one progresses in the mean field direction and the corresponding segments along which one progresses against the mean field direction (let N be the total number of such loop-backs). The averaging which leads to equation (4.6) is then carried out along the two sets of segments consecutively (with θ defined with respect to the mean direction of motion along each set of segments); and since the basic effect of looping back is just to increase the effective conduction path length, it is easily seen that this result must be corrected for the increase in actual path length traversed by electrons, e.g.,

$$\lambda_{\text{eff},\langle \boldsymbol{B}\rangle} = \lambda_{\text{mfp}}\langle \cos\,\delta\theta\rangle\,\frac{1}{N+1}\ , \qquad (4.7)$$

where N is the total number of loops (Fig. 1*c*). Naively, this might seem like a possibly large correction (since N would seem to be unconstrained and may be very large). However, in any reasonable model, in which the amount of tangling is bounded by the constraints on mixing velocities and on the typical scale lengths of unstable perturbations (whose motion relative to the background must be ultimately responsible for the tangling and loop-backs), N is in fact constrained to be of order kR, where k is the wavenumber of the dominant unstable modes (see § III) and R is the typical scale length associated with the halo as a whole; from the analysis of Malagoli, Rosner, and Bodo (1987), we may then conclude that $N \sim O(10)$, so that the magnitude of the additional correction is of the order of 10.

We have not as yet imposed any restrictions on the statistics of the magnetic field line random walk. It is somewhat difficult to translate the notion that the magnetic field is tangled into a quantitative statement, but let us assume for the moment that what is meant is that the probability distribution of $|\delta\theta|$ is highly concentrated near $\pi/2$; physically, this means that the local magnetic field is almost everywhere nearly perpendicular to the mean magnetic field (as in Fig. 1), a configuration which is consistent with (at least our notion of) tangled magnetic fields. In this case, it is plain from equation (4.7) that κ_{eff} will be much smaller than κ_0. How much smaller? In order to be more specific, let us adopt the simple but extreme case $\boldsymbol{B} = (\delta B, 0, B_0)$ in conventional Cartesian coordinates, with B_0 constant, $\langle |\delta B|\rangle \gg B_0$, and δB governed by Gaussian statistics (the second condition guarantees that $|\delta\theta|$ is almost always near $\pi/2$, while the third guarantees, among other things, that $\langle \delta B\rangle = 0$ and that B_0 is the mean field strength); this simple case is only illustrative, but retains all the essential physics which enters in the more involved spherical geometry appropriate to the halo problem (in which the mean field B_0 is radial). In that case, it is straightforward to show that

$$\langle \cos\,\delta\theta\rangle \approx [2\pi\langle (\delta B/B_0)^2\rangle]^{-1/2}\ \ln\,(2\langle (\delta B/B_0)^2\rangle)\ . \qquad (4.8)$$

Thus, for the somewhat extreme case of $\langle (\delta B/B_0)^2\rangle \approx 10^2$, equation (4.9) predicts a five-fold reduction in the conductivity along the mean magnetic field (i.e., in the *z*-direction), while a further 10^2-fold increase in $\langle (\delta B/B_0)^2\rangle$ reduces the effective conductivity by a further factor of ≈ 5. Thus, even very extreme assumptions about $\langle (\delta B/B_0)^2\rangle$ lead to only very modest reductions in the effective thermal conductivity (indeed, in an accreting flow in which the mean radial inflow speed monotonically increases with decreasing radius, one would expect the consequent shear to *decrease* $\langle (\delta B/B_0)^2\rangle$; see Tucker and Rosner 1983).

Finally, let us assume that the magnetic field is initially well-ordered and that the subsequent tangling of the frozen-in magnetic fields occurs because of mixing motions within the accreting flow. If, as supposed by Fabian, Nulsen, and Canizares (1984), this mixing occurs because of buoyancy-induced motions (driven by thermal instabilities), then these motions will be largely radial and characterized by flow speeds of the order of the general local accretion flow speed (as pointed out by Fabian, Nulsen, and Canizares). Hence, any two points on a given field line can separate at most by a distance comparable to the radial displacement experienced during the infall time. This implies that the above assumptions on the magnitude of $\langle(\delta B/B_0)^2\rangle$ are extremely conservative overestimates and hence that even the associated very modest reductions in the heat flux along the mean magnetic field substantially overestimate the actual heat flux reduction.

To conclude, the various arguments summarized above demonstrate that the assertion by Stewart *et al.* (1984) that thermal conduction can be entirely neglected if magnetic fields are tangled is not well-founded (even under rather optimistic interpretations of the notion of tangled magnetic fields): *The increase in effective path length can be shown explicitly to lead to only slight changes in the temperature profile, and the change in the local heat flux due to the modified effective heat conductivity is similarly only slight.* Hence the halo models proposed by Stewart *et al.* (1984) and Fabian, Nulsen, and Canizares (1984), which assume total suppression of thermal conduction are physically inconsistent.

V. THE FINE-TUNING PROBLEM

The fine tuning of the boundary conditions, alluded to in § I, would indeed be a problem if thermal conduction were the only heat source. In the absence of fine tuning, thermal conduction would either make the halos virtually isothermal, as it apparently does in a number of sources (see Stewart *et al.* 1984); or it would reduce the accretion rate by at most a factor of the order of 3 (Bertschinger and Meiksen 1986), so that the problem of excessive accreted matter would still exist for many sources which are manifestly not isothermal.

However, if an additional heat source exists which is proportional to the mass accretion rate, $h = \mu\dot{m}$, then the fine tuning of the accretion rate could come about in a stable, self-regulating manner. This heating could be produced as a result of star formation in the accretion flow, leading to the production of supernovae or cosmic rays, or it could result from cosmic rays produced as a by-product of accretion onto a central black hole. In either case, the flow would be self-regulating: If radiation losses temporarily exceed the heat input, then pressure gradients would develop in the flow and the accretion rate would increase; the increased accretion rate should lead to increased heating which would act to reduce the accretion rate until a balance is obtained. If the heat input temporarily exceeds the radiation losses, then the accretion rate will decrease; this decrease will reduce the heating, which in turn allows for the accretion rate to increase until a balance is obtained.[1]

We therefore expect halos of giant elliptical galaxies to fall into three categories: (1) those with boundary conditions such that the halos are essentially isothermal due to the dominance of the effects of thermal conduction; (2) those with boundary conditions such that radiative cooling occurs in the central regions, giving rise to an accretion flow, which in turn gives rise to an extended heat source in these central regions that regulates the accretion rate at a low value (these sources should show other evidence of the heating process, such as a centrally located nonthermal radio source); and (3) halos in which radiative cooling occurs, but no central heat source exists to offset the radiative losses, so the accretion proceeds at the maximum allowable rate given by equation (1.1). These sources should exist, but in our opinion should be rare, as the accumulation of several $10^8\ M_\odot$ of gas in the central regions of a galaxy over the course of 10^8 yr must make its presence felt as the by-products of star formation or the accretion of matter onto a massive central object generate a heat source that dramatically reduces the accretion rate.

VI. SUMMARY AND CONCLUSIONS

We have presented arguments against the existence of massive cooling flows in the central regions of giant elliptical galaxies. In particular, we have shown that a consistent description of the physics must include the effects of thermal conduction in an analysis of these flows; when these effects are included, we can explain the observed density and temperature profiles with the rate of mass inflow reduced from the values quoted in the literature by about a factor of 3–10 in the case of M87. Further, if a very modest amount of the material in cooling flows produces heating through accretion onto a central black hole, this heating can regulate the cooling flow in a thermally stable fashion at mass accretion rates of only a few $10^{-2}\ M_\odot\ \mathrm{yr}^{-1}$ This hundredfold suppression of the mass accretion rate avoids one of the major difficulties of cooling flow models, namely the whereabouts of the 10^9–$10^{12}\ M_\odot$ of matter that would have accreted in a cooling time. These arguments have been made earlier (see Tucker and Rosner 1985; Silk *et al.* 1986), but here we have focused on both a detailed discussion of why the neglect of thermal conduction is difficult to justify physically and the thermal stability of a gas in the presence of cosmic-ray heating.

The existence of radio sources in the central regions of most radiatively regulated halos indicates that a vigorous heat source capable of suppressing a cooling flow is indeed present in most cases. There may be some sources in which, for one reason or another, no heating occurs, and a cooling flow can develop at the maximum rate allowed by thermal conduction. However, such sources should be rare: this phase should be short-lived as star formation, supernovae, and accretion onto central massive objects should, over the course of a few 10^8 yr produce extensive heating that shuts off the cooling flow. In conclusion, the existence of temperature drops and excess central densities in the central regions of galaxies points to the importance of radiative losses, but not necessarily to the existence of massive cooling flows.

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[1] A rather similar issue arose a number of years ago in the modeling of the solar X-ray corona. By integrating the mass, momentum, and energy conservation equations for a spherically symmetric, thermally highly conductive corona *inward* from the corona to the base of the solar atmosphere, Moore and Fung (1972) discovered an extreme sensitivity of the solutions to the exact values of the coronal boundary conditions: Only for a very narrow choice of coronal boundary conditions was it possible to successfully integrate the equations to the base of the atmosphere; this, it was argued, suggested an exquisite balance of the energetics and hence an extreme tendency to instability. This stiffness of the equations, and the apparent fine tuning of coronal conditions necessary for successful solutions, turned out to be a complete artifact of the method of solution: Not only are the equations of motion not stiff when integrated outward, from the base to the hot corona, but furthermore the coronal atmosphere turns out to be largely self-regulating; for example, increases in the coronal heating rate naturally lead to "evaporation" of cool material at the coronal base, which increases coronal densities and hence coronal radiative losses, thus ultimately balancing the increased heating. Interestingly enough, it turns out that thermal conduction plays a central role in this self-regulating process, as increased (decreased) coronal temperatures lead to an increased (decreased) thermal heat flux at the coronal base and hence to an increase (decrease) in the supply of energy required to evaporate relatively cool base material (see Rosner, Tucker, and Vaiana 1978).

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