

The conduction equation can be written as:

$$\frac{\partial e}{\partial t} = -\nabla \cdot q$$

where energy is:

$$e = \frac{3}{2} n k_B T$$

energy flux is:

$$q = \kappa \nabla T$$

the conductivity can be written as:

$$\kappa_{sp} = n k_B \chi_c$$

where

$$\chi_c = 8 \times 10^{31} \left( \frac{T}{10 \text{ keV}} \right)^{5/2} \left( \frac{n}{5 \times 10^{-3}} \right)^{-1}$$

we can then compute the time for conduction front to cross one grid, that is:

$$dt = \frac{3dx^2}{2\chi_c} = \frac{ndx^2}{aT^{5/2}}$$

where n is measured in 10<sup>9</sup> per cc, T is in kelvin, dx is in cm, a is constant 1.8423.

Meanwhile, hydro time step can be calculated as:

$$dt = dx / c_s = \frac{dx}{\sqrt{\frac{\gamma k T}{m}}}$$

we can thus calculate the ratio of these two time steps (conduction over hydro):

$$r = A dx \frac{n}{T^2}$$

where A is a constant that takes value 1e-5 (this ratio is further divided by the CFL# we usually consider: 0.4). Here n is in per cc, T is in Kelvin, dx is in cm. Notice the relation between this ratio and the hydro parameters: r increases with n, decreases with T.

For the plasma conduction front problem (temperature 1e+6, density 1e+10, length scale 1e+6), this ratio is around 0.1, for small scale plasma physics problem, this ratio is around 10e-12, for big clump problem with temperature 1e+5, density 1e+1, length scale 1e+16, this ratio is around 100, for small clump problem with temperature 1e+4, density 1e+2, length scale 1e+11, this ratio is around 1, for large scale astrophysics problem, this ratio is around 1000.

Therefore it is necessary to develop two solvers for large r (explicit) and small r (implicit).