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THE EARLY EVOLUTION OF SUPERNOVA REMNANTS IN A HOMOGENEOUS MEDIUM: THE EFFECTS OF ELECTRON THERMAL CONDUCTION

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ABSTRACT

The role of electron thermal conduction in the evolution of supernova remnants may be fully understood only in the context of a two-fluid model, in which electron and ion temperatures are distinguished. During the early stages of supernova remnant evolution, high electron temperatures within the remnant imply that time scales for energy exchange by Coulomb collisions are long. Within the blast wave shock, electron and ion temperatures may be equalized by plasma instabilities, but in the interior of the remnant they may differ substantially. Only electron temperature profiles are fully smoothed by thermal conduction; ion temperatures in the interior of the remnant may remain high.

Electron thermal conduction through the blast wave shock itself may be suppressed by magnetic field effects and by plasma instabilities. We suggest that this is required if the remnant is to maintain approximate spherical symmetry while expanding in an ordered magnetic field.

We present two-fluid numerical simulations of remnant evolution in a homogeneous medium which illustrate the importance of these effects. Density profiles within the remnant are intermediate in form between those of nonconductive one-fluid simulations and those obtained in the isothermal blast wave similarity solution of Solinger *et al.* The radius of the remnant differs from that of the Sedov solution by less than a factor of 1.08. We briefly discuss the applicability of theory to observed remnants and discuss some possible tests. We emphasize that the discovery of iron emission lines in young remnants does not necessarily imply that the spectra should be interpreted as bremsstrahlung from a *Maxwellianized* electron distribution, but may be due instead to a suprathermal electron population.

Subject headings: hydrodynamics — nebulae: supernova remnants — shock waves

I. INTRODUCTION

The importance of thermal conduction in the evolution of supernova remnants has recently been emphasized by both Chevalier (1975) and Solinger, Rappaport, and Buff (1975). Solinger *et al.* derived a similarity solution under the assumption that electron and ion temperatures are equal and uniform throughout the interior of the remnant. The solution for the blast wave radius in this case is similar in functional form to the Sedov solution for adiabatic evolution (e.g., Woltjer 1972):

$$R_s = 1.17\left\{\frac{E}{\rho}\right\}^{1/5} t^{2/5}\,, \qquad (1)$$

where E is the energy of the remnant, ρ the interstellar density, and r the age of the remnant. However, the more efficient redistribution of energy in the interior results in this radius being larger by a factor of 1.08. The density distribution in this case is also smoother than in the adiabatic similarity solution, and the result is a smoother surface brightness profile.

In an alternative approach, Chevalier (1975) derived numerical solutions for the case in which electron and ion fluids are assumed to be at equal temperatures and the electron heat flux for an ionized plasma is given by the classical form (Spitzer 1962)

$$q = -\kappa\frac{dT}{dr}\,, \qquad (2)$$

$$\kappa = 0.6\,10^{-6}\,bT^{5/2}\ \text{ergs cm}^{-2}\,\text{s}^{-1}\,\text{k}^{-1}\,,$$

where we have set the Coulomb logarithm $\ln\Lambda = 30b^{-1}$ characteristic of the densities and temperatures of interest. This solution predicts a substantially faster evolution of the remnant radius as a function of time in the initial stages than the Sedov solution of equation (1), since the electron heat flux given by equation (2) redistributes the energy much more rapidly than dynamical motions. The outer radius in this case may be expressed as a function of time as

$$R_s = 9.76\times10^4\,E^{5/19}\,n^{-7/19}t^{2/19}b^{2/19}\ (\text{cgs})\,. \qquad (3)$$

A more detailed understanding of the role of electron thermal conduction in young remnants requires consideration of a number of additional effects. In § II we shall show that electron and ion temperatures may not be equal within the remnant and that a two-fluid model is required to obtain the density profiles. We shall also show that large electron mean free paths imply that the classical conduction of equation (2) must be replaced by a saturated heat flux such as that derived by Cowie and McKee (1977): redistribution of the energy by thermal conduction is correspondingly reduced. We consider whether electron thermal conduction may carry energy ahead of the blast wave shock and suggest on the basis of both theoretical and observational considerations that conduction through the shock must be inhibited. Both effects combine to slow the evolution far below the solution suggested by Chevalier (1975).

In § III we present two-fluid numerical simulations of the remnant evolution. In the early stages, density profiles are intermediate between nonconductive one-fluid simulations and the isothermal similarity solution of Solinger, Rappaport, and Buff (1975); in the later stages of the evolution, our results approach the latter case. Applications of the theory are briefly discussed in § IV.

II. PHYSICAL MODEL

a) Hydrodynamic Approximation: The "Saturated" Heat Flux

Consideration of the effects of conduction on young remnants is complex, since mean free paths in the plasma are extremely large compared with remnant radii (λ, the mean free path for electron energy exchange,

$$\lambda = \frac{3^{3/2}(kT_e)^2}{4\pi^{1/2}ne^4 \ln \Lambda} \approx 10^4\, T_e^{\,2}/n \text{ cm}\,, \tag{4}$$

is of order 200 pc for $T_e = 5 \times 10^8$ K and $n_e = 4$ cm^{-3}). In particular, we must ask how adequate a hydrodynamic treatment of the problem is, and what the appropriate heat flux to use under these circumstances is.

Cowie and McKee (1977) have suggested, from consideration of solar wind observations, that plasma instabilities constrain the particle distribution functions to be sufficiently isotropic to justify use of the hydrodynamic equations, even when the mean free path is large in comparison with fluid scale lengths. In this limit, particle distributions are no longer Maxwellian, and the local temperature is defined by the root mean square velocity dispersion of the particles. Cowie and McKee also argued that, in this limit, the heat flux approaches the maximum value $5\phi_s\rho c^3$ which may be transported by the electrons (ρ is the density of the plasma, c the isothermal sound speed $\equiv (P/\rho)^{1/2}$ and ϕ_s is an unknown factor of order unity which depends on the particle distribution function, plasma instabilities etc.). To interpolate between this value and the hydrodynamic limit (eq. [1]) we use the heat flux

$$q = \pm\kappa T_e/(L_T + 4.6\,\phi_s^{-1}\lambda)\,, \tag{5}$$

where $L_T \equiv T_e/|dT_e/dr|$ and the sign of equation (4) is opposite to the temperature gradient. With this value for the heat flux, the spherically symmetric hydrodynamic equations,

$$\frac{d\rho}{dt} = \frac{-\rho}{r^2}\frac{\partial}{\partial r}(r^2 v)\,, \tag{6a}$$

$$\rho\frac{dv}{dt} = -\frac{\partial}{\partial r}(\rho c^2)\,, \tag{6b}$$

$$\rho\frac{dU}{dt} = -\rho l + c^2\frac{d\rho}{dt} - \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 q)\,, \tag{6c}$$

should now describe the fluid in both limits. In equations (6), d/dt is the convective derivative, U the internal energy per gram, and l the net power loss per gram; we assume the gas is fully ionized at all times. However, some caution must be applied in interpreting observations in terms of the temperature when mean free paths are long compared to scale lengths, as we shall discuss in § IV.

b) Electron-Ion Equilibration: Conduction through the Blast Wave

Gas swept up by the remnant is heated in a collisionless shock, the postshock ion temperature being determined by the jump conditions (e.g., Landau and Lifshitz 1959); for a strong shock in which ion-electron equilibration does not take place within the shock, the postshock ion temperature is

$$T_i = \frac{3}{8}\frac{m_i v_b^2}{k} = 29\, v_b^2\,(\text{km}^{-1}\text{s})\text{ K}\,, \tag{7}$$

where m_i is the average ion mass and v_b the velocity of the blast wave.

For a fully Maxwellian plasma, Spitzer (1962) gives an electron-ion energy equipartition time by Coulomb collisions:

$$t_{\text{eq}} \approx 0.15\left(\frac{m_i}{m_e}\right)\left(\frac{m_e}{kT_e}\right)^{1/2}\frac{(kT_e)^2}{ne^4 \ln \Lambda}\,. \tag{8}$$

This is long in young remnants, with $t_{\text{eq}} \approx 3 \times 10^5$ yr for $T_e = 10^8$ K and $n = 1$, compared with typical remnant ages of 10^2–10^3 yr. (In eq. [8], e is the electron charge, m_e the electron mass, and T_e the electron temperature.) Thus unless collisionless processes in the shock are effective in heating the electrons, the electron temperature is $T_e \sim (m_e/m_i)T_i$ (Shklovskii 1973). In this case electron conduction is of negligible importance in the very early remnant evolution.

McKee (1974) has given strong arguments that collisionless processes in very high Mach number shocks are effective in equilibrating the ion and electron temperatures. However, equilibration within the shock by plasma instabilities requires the presence of turbulent electromagnetic fields with potentials comparable to the ion kinetic energy of $\frac{1}{2}\, m_i v_b^2$, and such fields may also suppress electron thermal conduction through the shock.

Behind the shock, electron-ion equipartition times may remain long by comparison with the remnant age, and redistribution of the electron energy may not result in a corresponding redistribution of the ion energy. The absence of isothermalization between electron and ion fluids is particularly important with regard to dynamical effects. For single-fluid models in which electron and ion temperatures are assumed equal, conductivity produces extremely small pressure gradients in the interior of the remnant, which decrease rapidly as the remnant expands. When electron and ion temperatures are distinguished, the ions in the central regions initially remain at high temperatures (which are, however, smaller than those in the adiabatic solution) with correspondingly large pressure gradients, and dynamical effects in the interior may continue to be important.

c) *Magnetic Field Effects*

We implicitly assume in the use of equation (5) that electron thermal conduction is not inhibited by magnetic fields within the remnant. This assumption is reasonable in the interior of the remnant since interstellar magnetic fields are uniform on scale lengths (~ 100 pc) large compared with remnant sizes (Spitzer 1968), and cannot inhibit the conduction along the field line direction. Near the edge of the remnant, tangential field structures may inhibit the electron heat flux; the present calculations, which ignore magnetic field effects, represent a maximum estimate of the effects of conduction.

However, the magnetic field structure *external* to the blast wave does imply that conduction through the blast wave is probably inhibited by the effects discussed in § II*b*. Unimpeded conduction through the shock would result in anisotropies roughly 5 times larger in the direction parallel to the field lines than perpendicular to the field lines, because of the faster propagation of energy in the parallel direction. Such an interpretation would conflict with X-ray observations of young remnants (Fabian, Zarnecki, and Culhane 1973), and we shall assume subsequently that conduction through the blast wave does *not* take place.

III. REMNANT EVOLUTION

a) *Numerical Method*

We have used a two-fluid beam scheme hydrodynamic code to numerically investigate the evolution of the remnants under the approximations described in § II. The code has been more fully described elsewhere (Cowie 1976); in the one-fluid approximation it reproduces previous remnant calculations both with and without classical thermal conductivity (Chevalier 1974, 1975). The gas is assumed fully ionized throughout, while radiative losses are included using the cooling function of Raymond, Cox, and Smith (1976); the rate of electron-ion energy exchange is given by the time scale of equation (8). Both these procedures are approximate since the electron and ion fluids are not Maxwellian in the early stages; however, the error introduced by this approximation is not expected to be large. Shocks are smoothed over several cells by the linear ion viscosity included in the code (typically 100 cells are in use at any given time). The position of the shock was chosen to be the maximum of the velocity profile. Electron-ion energy equipartition was enforced in this cell, and electron thermal conduction suppressed at its outer edge.

b) *Procedure*

To investigate the effects of varying the ambient density, the mass of material ejected by the remnant (the piston), and the conduction parameters, we ran a number of simulations. The supernova energy itself was held fixed and equal to 10^{51} ergs. The results may be generalized to arbitrary energy by a simple scaling law: if the energy is multiplied by d^{-2}, all densities by d, distances and times by d^{-1}, and masses by d^{-2}, then the form of the solutions is invariant. This scaling law, originally given by Chevalier (1974) for evolution with a classical thermal conduction, also applies to the more general case in which the heat flux is given by equation (5).

The energy of the remnant was initially deposited as kinetic energy in a uniform-density expanding shell of radius 2 pc. The velocity profile within the piston was chosen to be linear; Gull (1973) has shown that the properties of the later stages of evolution are relatively insensitive to the choice of density and velocity profile.

c) *Results*

For succinctness, we shall consider in detail only evolution in a uniform 1.0 cm^{-3} medium. We shall briefly summarize the other results subsequently.

Plots of the density, electron and ion temperatures, and the velocity as a function of radius at a number of times are shown in Figure 1. The plots are to some degree self-explanatory, and we shall only briefly outline the main physical points and contrast the solutions with those of Chevalier (1975) and Solinger, Rappaport, and Buff (1975).

In the initial stages of evolution the energy is in the form of kinetic energy; however, shortly thereafter the mass swept up is comparable to or greater than the piston mass and the energy is converted to thermal energy in the shock front. The energy is deposited equally between electrons and ions. In this stage, electron thermal conduction produces uniform electron temperatures but does not produce uniform ion temperatures since electron-ion exchange times are long. Thus the ion temperature distribution is similar to that obtained in adiabatic numerical simulations without conduction, modified somewhat by the ion conductivity and viscosity included within the code.

Beyond a radius of about 13 pc, electron temperatures have fallen sufficiently for electron-ion energy equipartition to take place throughout most of the outer regions of the remnant. Beyond this point, both ion and electron temperatures in the outer region of

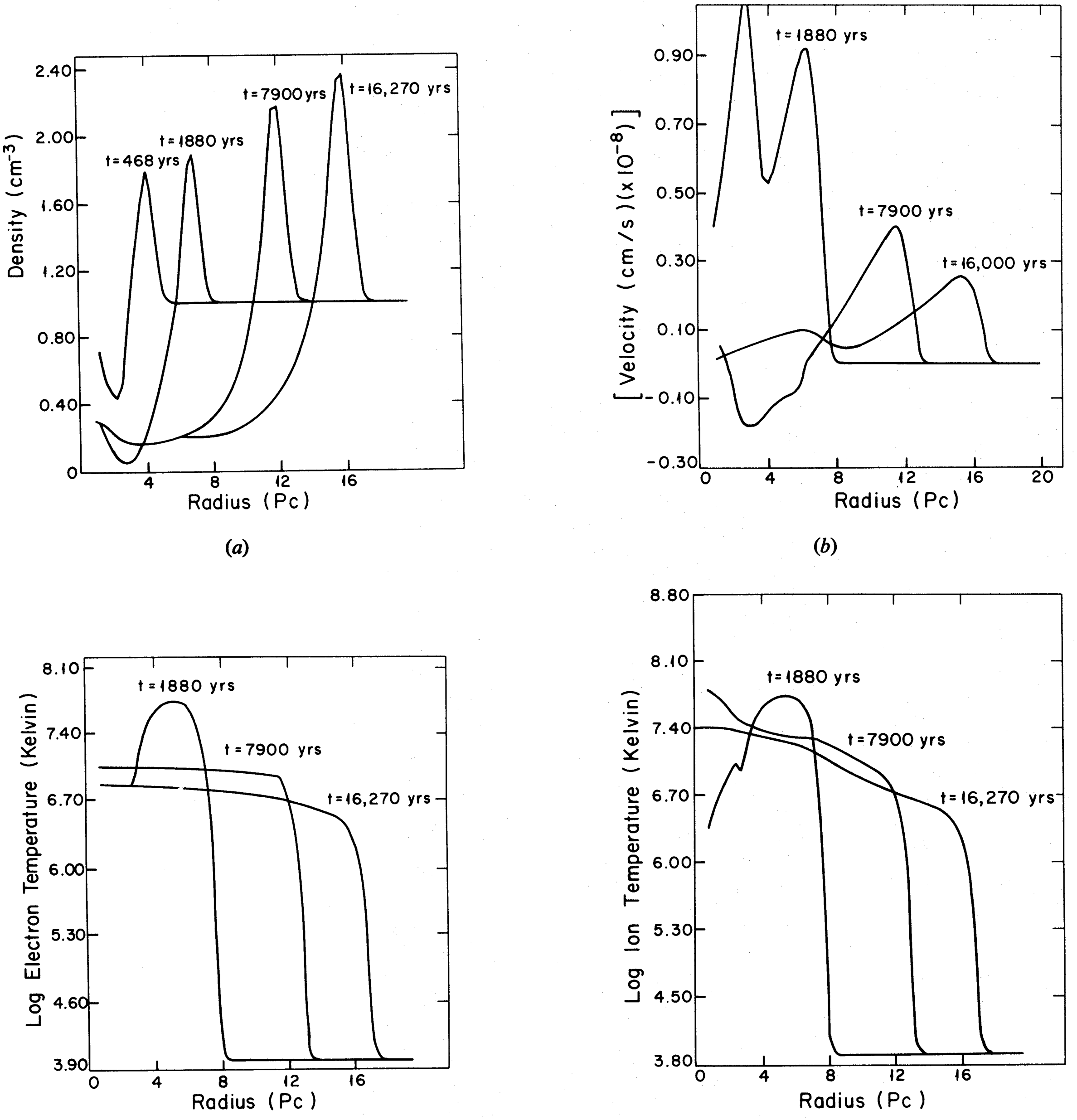

FIG. 1.—The properties of the remnant are plotted as a function of radius (pc) for the evolution of a 10^{51} erg explosion in an ambient density of 1 cm^{-3}; curves are labeled by age (yr). In Fig. 1*a* the density is shown in units of cm^{-3}; in Fig. 1*b*, the velocity in cm s^{-1}; in Figs. 1*c* and 1*d*, the electron and ion temperatures, respectively, in kelvins.

the remnant are uniform, and density profiles are similar to those of the isothermal blast wave solution (Solinger, Rappaport, and Buff 1975). We note that the post-blast-wave density increase ($\sim$2.3) is smaller than the adiabatic value (4) because of the effects of conduction, in the jump conditions and that the densities in the interior of the remnant are much larger than would be predicted by the adiabatic solution. Our results confirm that the isothermal self-similar solution should provide an adequate description throughout the later stages of the remnant lifetime; however, the position of the blast wave is not sufficiently determined for us to distinguish between the Sedov solution and the isothermal blast wave solution (1.08 times the adiabatic Sedov solution), for the remnant radius.

The remnant enters the radiative phase at a radius of about 16 pc. This is very similar to the radius found by Chevalier (1974, 1975) in his numerical simulations, and reflects the fact that radiation becomes important when the transport effects have ceased to be.

Variation of the conduction parameters had little effect on the solutions. However, some general points should be noted about the simulations in the lower densities. Lowering of the density results in an increase in the radius at which the ion temperatures become uniform and hence in the point at which the self-similar isothermal blast wave solutions becomes applicable. The radius at which this occurs is approximately given by

$$R = 13\left(\frac{E}{10^{51}\text{ ergs}}\right)^{2/7} n_0^{-3/7}\text{ pc}, \qquad (9)$$

which will be much larger than observed remnant radii if the density in which the remnant is evolving is less than 0.3 cm^{-3}. For smaller radii the solutions are intermediate in form between the adiabatic and isothermal blast wave solutions.

d) *How Sensitive Is the Model to the Assumptions?*

In view of the discrepancy between our results and those of Chevalier (1975), and of the fact that a number of our assumptions differ, it is important to ask which is responsible for the differences.

Clearly the assumption that thermal conduction is suppressed through the blast wave is sufficient *of itself* to guarantee that the solutions for the radius approximate the Sedov solution. However, even if this assumption is relaxed subject to the assumption of spherical symmetry, the upper bound on the energy transfer by electrons given by the saturated heat flux of equation (6) *continues to imply* that the solution is similar in functional form to the Sedov solution. The faster redistribution of energy in this case implies that the radius is about twice that of the Sedov solution but is still substantially smaller than the radius obtained from equation (2). The two-fluid calculation has little effect on the position of the outer radius of the remnant.

The form of the conductivity has little effect on the interior density and temperature distributions, since heat transfer implies approximate isothermalization of the electron fluid. Differences in ion temperature and density profiles within the remnant are principally due to the use of the two-fluid approximation in the present paper.

IV. DISCUSSION

We shall divide our discussion into two sections, and consider separately young remnants where electron-ion equipartition has not yet taken place, and older remnants where a one-fluid approximation is valid. The dividing radius between the two cases is given by equation (9). Typical examples of young remnants are Cas A, Tycho, and SN 1006, while remnants such as Cygnus and Vela may fall into the category of older remnants.

a) *Young Remnants*

In summary, we have emphasized that thermal conduction is unlikely to substantially affect the rate of evolution of the radius of young supernova remnants or to greatly affect the density profile within young remnants. Systematic temperature variations perpendicular to the magnetic field line structure are predicted; however, these may be obscured by local temperature fluctuations due to inhomogeneities (McKee 1974; McKee and Cowie 1975).

Electron mean free paths may be larger than the remnant size for very young remnants such as Cas A (this remains true even if temperatures derived from the observed X-ray spectrum are substituted for temperatures inferred from radial motions within the remnant); therefore, the electron distribution within the remnant may not be Maxwellian. Thus interpretation of the spectra of Cas A and Tycho on the basis of multitemperature thermal bremsstrahlung fits, together with iron emission lines, may not be entirely justified, and alternative fits should continue to be attempted *despite* the presence of the iron emission lines. We note that the spectrum of Cas A may be well fitted by a power law of index −4.5 (Serlemitsos 1973) (together with an iron emission line), and that of Tycho by two power-law spectra (together with an iron emission line); such a spectrum could be produced by a non-Maxwellian population of suprathermal electrons.

b) *Older Remnants*

In older remnants (as defined by eq. [9]), our results confirm those of Solinger, Rappaport, and Buff (1975) who have previously discussed the observational consequences and suggested that the results may ultimately be confirmed by the smoother surface brightness profile predicted by the thermal conduction models. We note, however, that thermal evaporation from embedded cold clouds may produce substantial temperature and density variations within the remnant, making this effect extremely difficult to detect.

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