RABBITS VS. SHEEP
A "CONTINUOUS" FIXED POINT PROBLEM

Jake Brock (Using Example from Steven Strogatz: Nonlinear Dynamics and Chaos)

POPULATION GROWTH

- Exponential growth rate (rate proportional to population size):

$$\frac{dN}{dt} = rN$$

- Larger populations compete for resources. Let's say reproduction (dN/dt) per capita (N) decreases with population size (N):

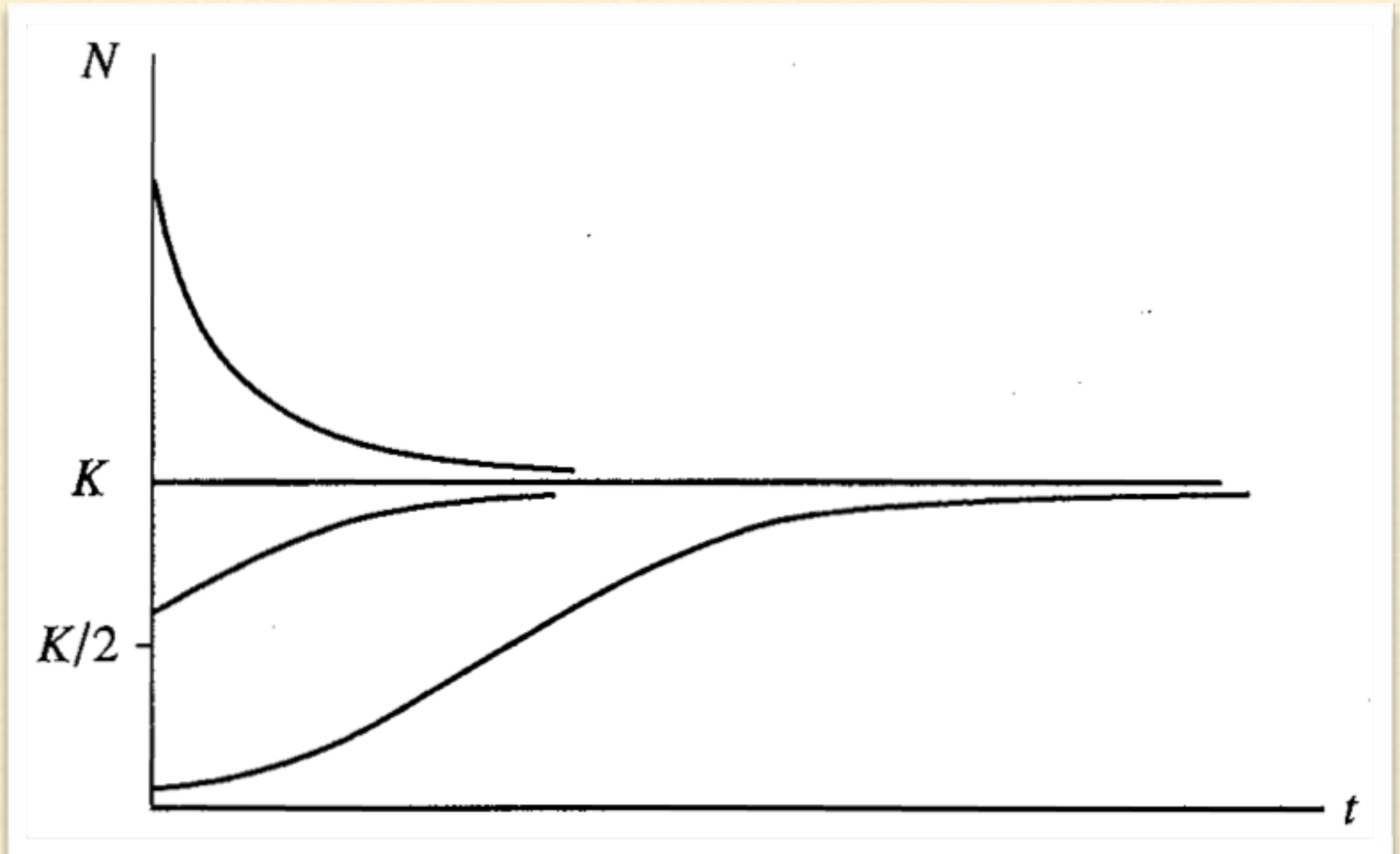
$$\frac{dN}{dt}/N = r - \frac{r}{k}N$$

- **The Logistic Equation** $\longrightarrow$ $\boxed{\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right)}$

The Logistic Equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{k}\right)$$

Carrying Capacity



Works: Bacteria, yeast, simple organisms

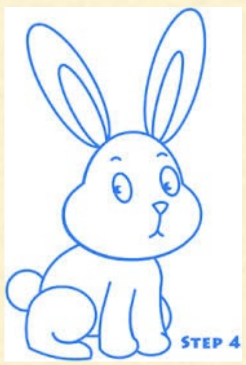
Doesn’t Work: Fruit flies, flour beetles, complex life cycles

RABBITS VS. SHEEP

- Both eat grass.
- Sheep usually beat rabbits when fighting for same tuft of grass.
- Start with logistic equation for each, but reduce growth in proportion to competitor's population size.

RABBITS VS. SHEEP

Fast reproduction. Higher carrying capacity. Sheep threaten them.



$$\frac{dR}{dt} = r_R R\left(1 - \frac{R}{k_R} - aS\right)$$

r_R	3
k_R	300
a	2/300

Slower reproduction. Lower carrying capacity. Rabbits threaten less.

$$\frac{dS}{dt} = r_S S\left(1 - \frac{S}{k_S} - bR\right)$$

r_S	2
k_S	200
b	1/200

RABBITS VS. SHEEP

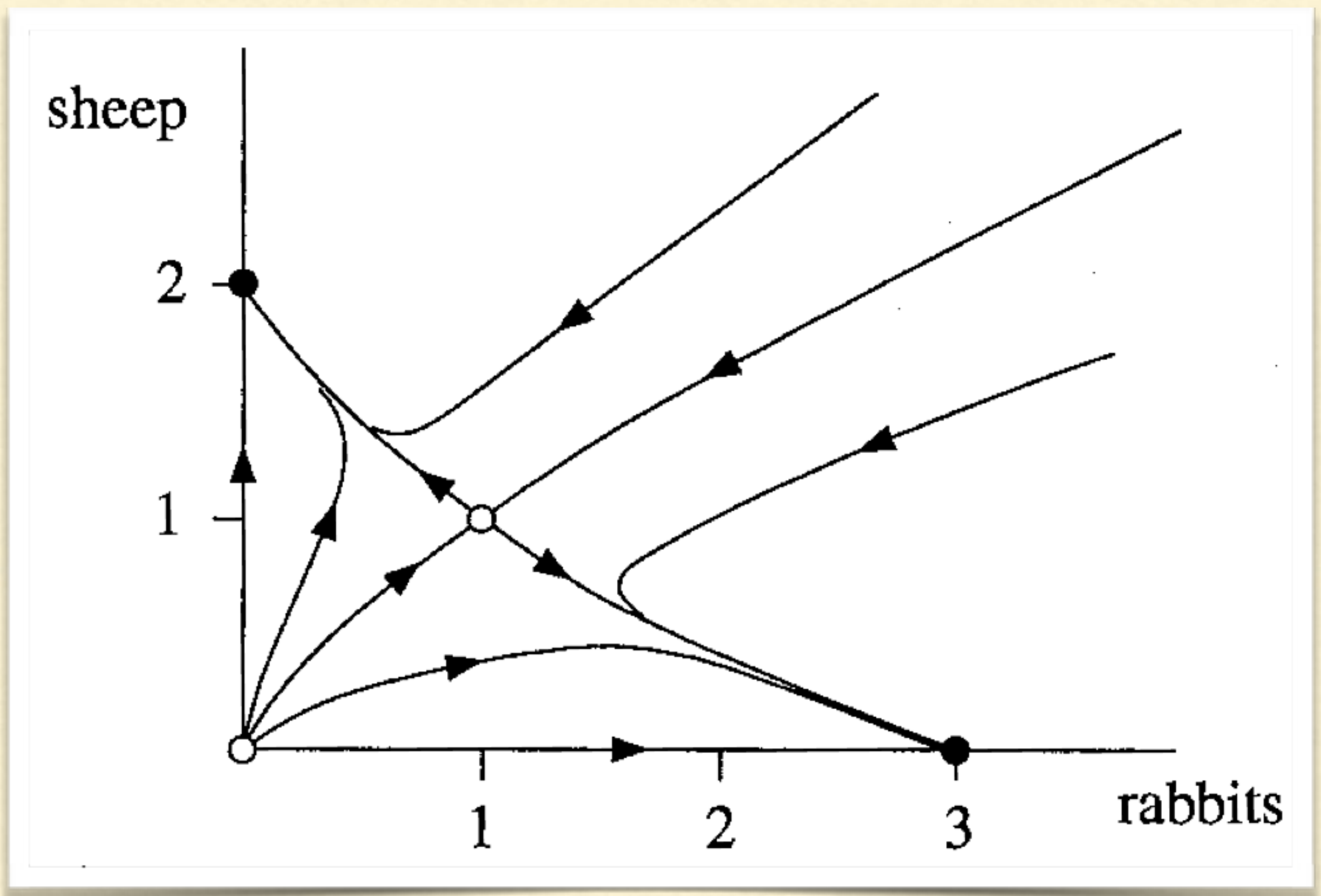
$$\frac{dR}{dt} = r_R R \left(1 - \frac{R}{k_R} - aS\right)$$

$$\frac{dS}{dt} = r_S S \left(1 - \frac{S}{k_S} - bR\right)$$

Fixed points where dR/dt = dS/dt = 0:

RABBITS	SHEEP
0	0
0	k_S = 200
k_R = 300	0
100	100

RABBITS VS. SHEEP



Fixed points where $dR/dt = dS/dt = 0$:

RABBITS	SHEEP
0	0
0	k_S = 200
k_R = 300	0
100	100

THANK YOU!