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Effect of a vertical magnetic field on turbulent Rayleigh-Bénard convection

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The effect of a vertical uniform magnetic field on Rayleigh-Bénard convection is investigated experimentally. We confirm that the threshold of convection is in agreement with linear stability theory up to a Chandrasekhar number $Q \simeq 4\times10^6$, higher than in previous experiments. We characterize two convective regimes influenced by MHD effects. In the first one, the Nusselt number Nu proportional to the Rayleigh number Ra, which can be interpreted as a condition of marginal stability for the thermal boundary layer. For higher Ra, a second regime $\mathrm{Nu}\sim\mathrm{Ra}^{0.43}$ is obtained.

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We consider the Rayleigh-Bénard problem of a fluid layer heated from below and cooled from above. Convection occurs for a Rayleigh number [1] higher than a critical value Ra_c. We use an electrically conducting fluid (mercury) and impose a uniform vertical magnetic field B, resulting in eddy currents and flow damping by the Lorentz force. Energy is then dissipated by the Joule effect in addition to viscosity. As a consequence, the convection threshold increases with the magnetic field, as predicted by the linear stability analysis of Thompson [2] and Chandrasekhar [3,4]. We find a good agreement with this linear theory, confirming the experiments of Nakagawa [5,6] and Jirlow [7]. However, our main result is a characterization of turbulent magnetohydrodynamics (MHD) regimes occurring beyond the threshold. This problem is of interest for technical applications, as in the control of crystal growth, and for the convection in planetary cores or stars. Moreover this study can shed light on ordinary convection, providing a means of external action on the system. As pointed out by Bhattacharjee *et al.* [8], the different regimes of ordinary turbulent convection have an MHD counterpart for which the scaling laws of the different regimes can be more sharply distinguished.

The effect of the magnetic field is classically estimated by the Chandrasekhar number Q, ratio of the damping times by Joule and viscous effects respectively,

$$Q=\frac{B^2\sigma L^2}{\rho\nu}, \tag{1}$$

where σ is the electrical conductivity and ρ the mean fluid density. In our experiment, $\sigma=1.04\times10^6\ \mathrm{m}\Omega^{-1}$ and $\rho=1.36\times10^4\ \mathrm{kg\,m^{-3}}$, so $Q=3.21\times10^7\,B^2$, with B in T. Note that Q is the square of the Hartmann number, initially introduced for duct flows. The magnetic diffusivity $\eta=0.8\ \mathrm{m}^{-2}$ is much larger than the thermal diffusivity, by a factor 2×10^5. As a consequence, the magnetic field produced by eddy currents can be neglected with respect to the imposed field B.

The convection threshold Ra_c increases with the magnetic field, and $\mathrm{Ra}_c\rightarrow\pi^2 Q$ when $Q\gg1$. This limit corresponds to negligible viscosity effects, and is independent of the boundary conditions for velocity (free slip or no slip). Note also that the threshold does not depend on the electric boundary conditions (i.e., wall conductivity), whatever the value of Q.

Our experimental cell [9,10] is a vertical cylinder with aspect ratio $\Gamma=D/H=1$, suitable to reach high Rayleigh numbers for a given available heating power. Due to the confinement by the lateral walls, the geometry differs from the standard Rayleigh-Bénard problem and from Nakagawa's experiments (with thickness 3 to 6 cm and large aspect ratio). In the absence of magnetic field, this difference results in a higher convection threshold. However, the influence of the lateral confinement is very weak in the turbulent regime. The system is then mainly controlled by the thermal boundary layers, much thinner than the diameter. In the presence of a vertical magnetic field the most unstable modes have a small horizontal wavelength (in $Q^{-1/3}$), so we expect that the lateral confinement is not significant even near the instability threshold. The cell is introduced in an electromagnetic coil, producing a vertical uniform magnetic field, tunable from 0 to 0.4 T, with uniformity better than 10^{-3}.

In summary the dynamics is in principle determined by five nondimensional numbers, the Rayleigh number Ra, the Chandrasekhar number Q, the Prandtl number $\mathrm{Pr}=\nu/\kappa$, the magnetic Prandtl number η/κ, and the aspect ratio Γ. We vary Ra up to 3×10^9 and Q up to 4×10^6, while the last three parameters are fixed, with $\mathrm{Pr}=0.025$, $\eta/\kappa=2\times10^5\simeq\infty$, and $\Gamma=1$ (but we expect that Γ has little influence).

The bottom plate is in copper (coated with nickel to protect it from mercury) and is heated by an electrical resistor, shaped in a double spiral to avoid generation of a magnetic field. The top plate, also in copper, is cooled by a water circulation, and is regulated in temperature (measured on the axis of the cell, at 3 mm above the mercury layer). The lateral wall is a 2-mm-thick stainless steel cylinder. The cell is thermally insulated from the outside by neoprene layers, with a total loss coefficient 0.2W/K.

The temperature is measured in each plate by eight thermistors, equally spaced in azimuth, at 5 cm from the cell axis, and 3 mm from the plate surface in contact with mercury. The temperature difference Δ between top and bottom is obtained as an average over these probes. In each experiment, we set the heating power and the temperature of the top plate (in such a way that the mean temperature of the cell is close to the coil temperature, to minimize lateral heat

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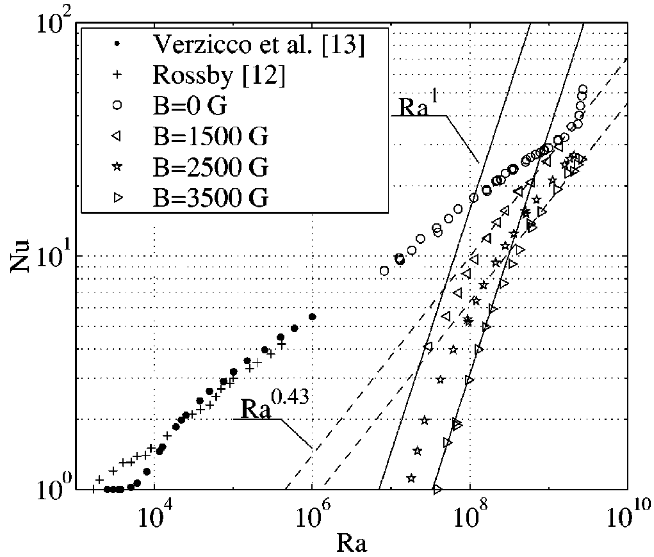

FIG. 1. Nu vs Ra with and without magnetic field. Without magnetic field: (·) data of Rossby (1968) [12], (+) numerical simulations of Verzicco and Camussi (1996) [13], (○) data of Cioni *et al.* [9]. With magnetic field, present experiment: (◁) at $Q=7.22\times10^5$ ($B=1500$ G), (*) at $Q=2.0\times10^6$ ($B=2500$ G), and (▷) at $Q=3.93\times10^6$ ($B=3500$ G). The two dashed lines represent the $\mathrm{Ra}^{0.43}$ scaling and the solid ones represent the Ra^1 scaling.

loss). Then we measure the mean temperature difference Δ in the permanent regime, and deduce the Nusselt number [11] Nu, ratio of the heat flux to the purely diffusive one. Our main experimental result in this Rapid Communication is the determination of Nu as a function of the two parameters Ra and Q. We also measure local temperature using a probe movable along the cell axis, providing vertical temperature profiles and statistics of local fluctuations.

We first recall in Fig. 1 the results Nu versus Ra in the absence of a magnetic field (in the same cell). The results of Rossby [12] and Cioni *et al.* [9] both fit very well with a law $\mathrm{Nu}=0.14\mathrm{Ra}^{0.26}$, in the range $10^4\leqslant\mathrm{Ra}\leqslant4.5\times10^8$. Numerical results [13], performed in the same cylindrical geometry as in our experiment, confirm this law for $\mathrm{Ra}\geqslant2\times10^4$ (the numerical results at lower Ra differ from the Rossby experiments due to the different aspect ratio). Takeshita *et al.* [14] found the same scaling in Ra, although their prefactor is larger by about 20%. Transitions to new regimes [9] are obtained for $\mathrm{Ra}\geqslant4.5\times10^8$.

The effect of a magnetic field reduces heat transfer as expected. With a magnetic field of 0.35 tesla, we are able to totally suppress convection, reaching Nu=1. The threshold is consistently obtained both by the change in the vertical temperature profile (which is linear in the diffusive regime), and by the onset of temperature fluctuations. No hysteresis is observed, in agreement with nonlinear stability analysis: no subcritical convection is expected in our limit of large magnetic diffusivity [15]. The threshold fits well with the theoretical prediction, as shown in Fig. 2. The temperature difference is $\Delta=1.15$ °K and heating power 1.5 W. This is close to the minimum value for reliable measurements with our experimental cell, so we are not able to study the convection threshold for smaller values of the magnetic field.

Beyond the threshold, we observe a turbulent regime, which we call regime I, with a scaling law Nu∼Ra, which

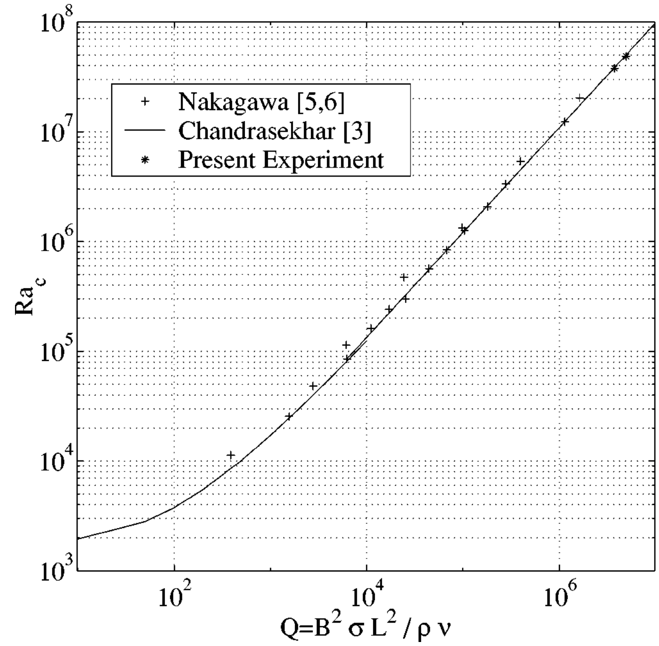

FIG. 2. A comparison of the experimental and theoretical results on the critical Rayleigh numbers for the onset of instability. The theoretical (Ra_c, Q) relation is shown by the solid line curve. (+) are the experimentally determined points of Nakagawa (1957) [5,6] and (*) are the experimental findings of the present work.

covers one decade at the highest magnetic field, 0.35 T. We can understand this result by an argument of marginal stability of the thermal boundary layer, in the spirit of Malkus [16] and Howard [17]. Turbulence is assumed to mix the temperature in the interior, confining the gradient to a thermal boundary layer near the walls. The thickness λ of this boundary layer is assumed to be maintained at the limit of stability. In other words, the Rayleigh number Ra_λ based upon this thickness remains equal to the critical one, $\mathrm{Ra}_\lambda\sim\mathrm{Ra}_c=\mathrm{const}$. In the absence of a magnetic field this yields the classical law $\mathrm{Nu}\sim(\mathrm{Ra}/\mathrm{Ra}_c)^{1/3}$. Here Ra_c depends on the magnetic field. From the Chandrasekhar theory $\mathrm{Ra}_c\sim\pi^2 Q_\lambda$ for large Q_λ. Furthermore, using the fact that $Q_\lambda\sim Q\lambda^2/L^2$ and $\mathrm{Nu}\sim L/\lambda$ we deduce [8]

$$\mathrm{Nu}\sim\frac{1}{\pi^2}\frac{\mathrm{Ra}}{Q}\quad\text{regime I.}\tag{2}$$

Therefore, this law is the MHD counterpart of the classical $\mathrm{Ra}^{1/3}$ law. We check that the dependence in Q is consistent with Eq. (2), when comparing the results at $B=0.25$ T and $B=0.35$ T (Fig. 1).

The $\mathrm{Ra}^{1/3}$ scaling is well verified [18] in ordinary convection at large Prandtl number but not at moderate or small Prandtl numbers. We can understand that the MHD case at large Q is akin to ordinary convection at large Pr. Indeed, the Joule damping can be viewed as a kind of "magnetic viscosity" [2,4], with value $\nu_B=B^2\sigma L^2/\rho$, much higher than the viscosity ν, and the "Prandtl number" $\nu_B/\kappa=Q$ Pr is large. Although this argument brings physical insight, it must be taken with caution however, as the Joule damping is anisotropic and acts equally at all scales, unlike viscosity.

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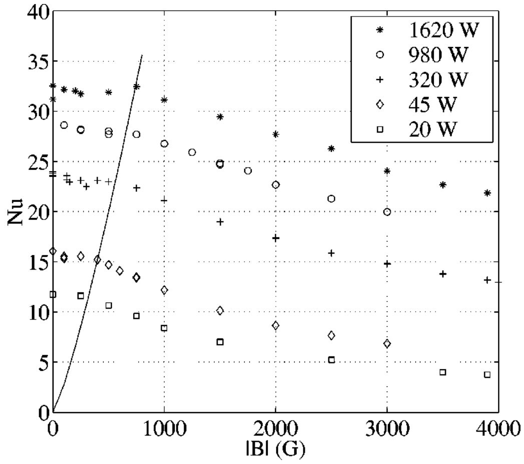

FIG. 3. Plot of Nusselt vs magnetic field B at fixed heating power for five different heating powers (*) 1620 W, (○) 980 W, (+) 320 W, (◇) 45 W, (□) 20 W. The solid line represents the curve of constant interaction parameter $N_0=2$ $[\mathrm{Nu}=9.94\times10^{-3}(B^2)^{0.613}]$ beyond which the damping of convection becomes important.

At higher Ra, the slope of the curve Nu versus Ra becomes weaker, and a power law $\mathrm{Nu}\sim\mathrm{Ra}^{0.43}$ seems to emerge, and we call it regime II. Notice that this is in reasonable agreement with an adaptation [8] to the MHD case of the mixing zone model [19], involving a matching of the thermal boundary layer with the mixed core by thermal plumes. This model leads to the $\mathrm{Nu}\sim\mathrm{Ra}^{2/7}$ scaling in the absence of a magnetic field, and to the scaling $\mathrm{Nu}\sim\mathrm{Ra}^{1/2}/Q^{3/4}$ with a magnetic field. Then thermal plumes are damped by the Joule effect rather than viscosity. Although this model is in reasonable agreement with our experimental law $\mathrm{Nu}\sim\mathrm{Ra}^{0.43}$, the predicted dependence in Q is too strong. Comparisons of the curves at 0.25 and 0.35 T in Fig. 1 is rather consistent with

$$\mathrm{Nu}=0.1\mathrm{Ra}^{0.43}Q^{-0.25}\quad \text{regime II.} \tag{3}$$

The crossover with Eq. (2) occurs at $\mathrm{Nu}\sim0.10Q^{0.32}$, so that the range with marginal stability behavior (2) (from Nu=1 to this value) indeed increases with the magnetic field.

We now study in more detail the dependence in Q (magnetic field) for intermediate cases (regime III). We perform a set of experiments, measuring the temperature difference Δ for a given heating power and for successive values of the magnetic field (we then travel along a line NuRa=const on the Nu-Ra plot of Fig. 1). We plot $\mathrm{Nu}\sim1/\Delta$ versus the magnetic field in Fig. 3.

There is a very weak decrease of a few percents followed by a plateau, and then a strong decrease. The first weak decrease seems related to the braking of the global convective circulation occurring in the absence of the magnetic field [9]. This global circulation induces a temperature perturbation on the bottom and top plates with a dipolar azimuthal dependence. Furthermore it induces an oscillation in temperature signals at frequency $f_p\simeq U/L$ proportional to the velocity U. We find that the dipolar temperature structure on the plate is reduced while the frequency decreases for increasing magnetic field and the oscillation eventually disappears for a field of a few hundred gauss. This damping is effective when the

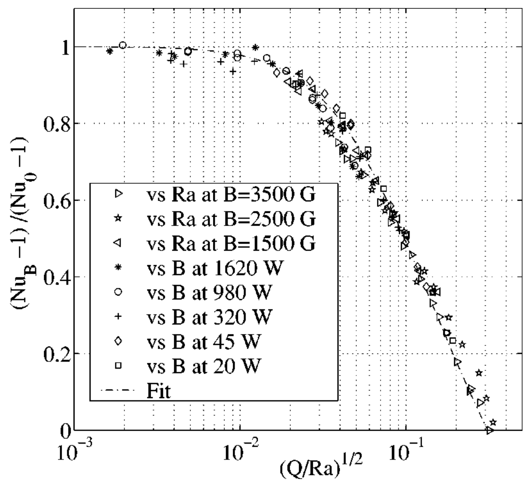

FIG. 4. Normalized Nusselt number Nu* vs $(Q/\mathrm{Ra})^{1/2}$. The dashed line represents the empirical fit given in Eq. (4).

Joule dissipation time is of the order of the turnover time L/U, i.e., the interaction parameter $N=(\sigma B^2/\rho)(L/U)=Q/\mathrm{Re}$ takes a given value N_0 of order one. Using the velocity experimentally estimated in [9] in the absence of magnetic field, this condition becomes $Q\simeq N_0\mathrm{Re}\simeq12.0N_0\mathrm{Ra}^{0.424}$. Using the experimental relation $\mathrm{Nu}=0.14\mathrm{Ra}^{0.26}$, this can be also written as $\mathrm{Nu}=0.030(Q/N_0)^{0.613}$. This condition, represented in Fig. 3, provides indeed a good estimate for the disappearance of the global circulation (taking $N_0=2$).

The strong decrease of Nu occurs beyond this point. We characterize this decrease by the reduced Nusselt number $\mathrm{Nu}^*=(\mathrm{Nu}_B-1)/(\mathrm{Nu}_0-1)$, where Nu_0 is the Nusselt number in the absence of magnetic field for the same Ra. This quantity depends both on Ra and Q, but all our data come close to a single curve when we plot Nu* versus the ratio Ra/Q, as shown in Fig. 4. We have tried various parameters of the form Ra/Q^γ, and the collapse is optimum for $\gamma\simeq1$. The parameter Ra/Q can be interpreted as the Rayleigh number constructed with the magnetic diffusivity ν_B. We can

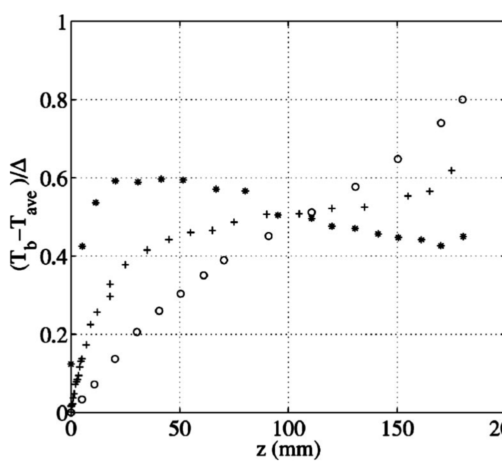

FIG. 5. Temperature profile of the time averaged temperature, over time 30 min, vs z corresponding to (○) the linear diffusive profile (NU=1), (+) regime I for $\mathrm{Ra}=1.9\times10^8$ at $B=3500$ G (corresponding to Nu=6), (*) regime II for $\mathrm{Ra}=3.9\times10^8$ at B=1500 G.

understand that this should be the relevant parameter when magnetic effects dominate viscous effect, so that Nu is a function of Ra/Q as in Eq. (2). However, it is not clear to us why Ra/Q should be the relevant parameter for Nu*. A similar representation has been proposed by Okada and Ozoe [20] in convection with a horizontal temperature gradient and a vertical magnetic field. A good collapse on a single curve was obtained with the parameter Ra/$Q^{3/2}$, while Lykoudis [21] earlier introduced the parameter Ra/Q^2. Both representations yield a poor collapse in our case. We find that the empirical law

$$\mathrm{Nu}^* = 1 - \left[0.81 + 5.9\times10^{-2}\left(\frac{Q}{\mathrm{Ra}}\right)^{-1/2}\right]^{-2} \tag{4}$$

provides a good empirical fit to our data.

Vertical profiles of the mean temperature are plotted in Fig. 5. For Nu=1 we observe the linear diffusive profile. We check that the temperature gradient at the wall is in agreement with the heating flux, $H=-\kappa dT/dz \simeq \Delta/\lambda$. In regime I the thickness λ is controlled by the marginal stability of the thermal boundary layer while temperature becomes well mixed in the interior. In regime II, we find remarkably that the interior becomes stably stratified $(dT/dz>0)$. This probably means that convection is blocked in the bulk but persists near the lateral walls from which horizontal motion could maintain the stable stratification. Stable stratification in the interior of a convective cell has been also reported in the absence of magnetic field (see, e.g., [14]), but the effect here is much stronger.

In conclusion, we can distinguish three turbulent MHD regimes:

(i) Regime I is characterized by the marginal stability condition in the boundary layers (setting the boundary layer thickness) with a well mixed interior. The interaction parameter N is small, so that inertia is negligible with respect to the linear Joule effect. Temperature advection therefore remains the only nonlinear term, as in ordinary convection in the limit of high Pr.

(ii) Regime II is characterized by a heat flux (3), and by a stably stratified interior.

(iii) Weak magnetic field suppresses the global circulation when $N\sim1$, reducing Nu by only a few percents. This suggests that the global circulation has only a very small influence on heat transfer, in contradiction with some theoretical models. A significant drop in Nu occurs for a magnetic field about twice higher.

The mechanisms of heat transfer remain to be understood. In particular, the possible role of plumes is an intriguing question. Probability distributions of temperature fluctuations show no exponential tails in a strong magnetic field, by contrast with ordinary convection. This suggests an absence of plumes. Alternatively they may be expelled from the cell axis, where measurements are made.

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[1] The Rayleigh number is equal to $\mathrm{Ra}=g\alpha\Delta L^3/\nu\kappa$, where Δ denotes the temperature difference between the bottom and top plates, $L=0.213$ m the fluid layer thickness, $g=9.8$ ms^{-2} the acceleration of gravity, $\alpha=1.81\times10^{-4}$ the coefficient of thermal expansion, $\nu=1.08\times10^{-7}$ ms^{-2} the kinematic viscosity, and $\kappa=4.6\times10^{-6}$ ms^{-2} the thermal diffusivity. Sometimes the Grashof number Gr=Ra/Pr is used instead, where Pr is the Prandtl number equal to Pr=ν/κ.

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[11] The Nusselt number is defined from the heat flux H^* across the convective cell (electrical heating power divided by the horizontal section 356 cm^2) as $\mathrm{Nu}=H^*L/\kappa C_p\Delta$, where C_p $=139$ J K kg)$^{-1}$ is the volumetric heat capacity of mercury.

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