10. Magnetism

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1 Magnets

Everyone has seen magnets. The simpest kind is a bar magnet. The North poles of two bar magnets repel while the North and South Poles will attract. The Earth itself is a giant magnet, which is why the North pole of a bar magnet points North if suspended freely.

You can bend a bar magnet into the shape of a horse shoe so that the N and S poles are closer together. This produces a magnetic field that is strong and more or less uniform (constant) in the region between the poles.

It is not possible to get an isolated North or South pole: if you cut a magnet in two you will just get two bar magnets, the point where you cut it producing a N-S pair. Much of this was known even in medieval times.

2 Electric Currents Produce Magnetism

The surprising discovery of the nineteenth century was that electric currents produce magnetic fields. Thus a wire carrying carrying a current can attract or repel a magnet. Conversely, magnet exerts a force on a wire carrying a current.

Thus a charge at rest produces just an electric field. A moving charge produces both an electric and a magnetic field. Also, a charge q at rest has a force acting on it that is just given by $q\mathbf{E}$. When it is moving, the force is the sum of the electric and the magnetic force.

The magnetic force depends on the velocity of the particle as well as its charge and the magnetic field at its location: it is proportional to the product of the velocity and the magnetic field. How do you multiply two vectors (velocity and magnetic field) to get a third vector (force)? This is the notion of cross product which we will review.

3 Cross-Product of Vectors

Given two vectors $\mathbf{a}$ and $\mathbf{b}$ their cross product $\mathbf{a} \times \mathbf{b}$ is defined as the vector whose magnitude is

$$ab \sin \theta$$

where θ is the angle between them. The direction of $\mathbf{a} \times \mathbf{b}$ is given by the right hand rule:

Point the index finger of your right hand along $\mathbf{a}$.

The middle finger along $\mathbf{b}$.

Stretch out your thumb. It points in the direction of $\mathbf{a} \times \mathbf{b}$.

Note that the direction of the product is reversed if we switch $\mathbf{a}$ and $\mathbf{b}$:

$$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}.$$

Thus, if $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the unit vectors along the x, y, z axes,
$\mathbf{i} \times \mathbf{j} = \mathbf{k}, \quad \mathbf{j} \times \mathbf{k} = \mathbf{i} \qquad \mathbf{k} \times \mathbf{i} = \mathbf{j}$
But
$\mathbf{j} \times \mathbf{i} = -\mathbf{k}, \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i}, \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j}$
The cross product of any vector with itself is zero.

$$\mathbf{a} \times \mathbf{a} = -\mathbf{a} \times \mathbf{a} = 0.$$

4 Definition of Magnetic Field

Consider a wire carrying an electric current I. A horse shoe magnet is placed so that the line from the North to the South pole is perpendicular to the wire. There will then be a force acting on the wire in the direction perpendicular to both the wire and the N-S line of the magnet.

If you hold the fingers of your right hand so that the index finger points along the current and the bent fingers point from the N pole to the S pole of the magnet, the force will point along your thumb.

The magnitude of the force felt by the wire is proportional to the current I it carries, as well as its length L. We define the magnitude B of the magnetic field at the position of the wire as the force divided by the current.

$$F = ILB.$$

The magnetic field is a vector: it points from the N pole to the S pole of the magnet.

What if the wire is at an angle θ to the magnetic field? Then the magnitude of the force is

$$F = ILB \sin \theta.$$

If the wire is parallel to the magnetic field there is no force.

The direction of the force is given by the right hand rule: index finger of right along the current, the middle finger along the magnetic field and the thumb along the force.

Thus we can write

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B} \tag{1}$$

as the definition of the force. The direction of the length vector is along the wire, in the direction of the current.

Remember by the way that the direction of the current is from the positive end of the battery producing it to the negative end.

5 The Force On a Moving Charge

The above law of force on a current-carrying wire can be thought of in another way:

The force on a charge q moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ is

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}. \tag{2}$$

After all, an electric current is nothing but the movement of electric charges. In some time t,the amount of charge that has moved through a wire carrying a current I is

$$q = It.$$

On the other hand, the average velocity of these charges is

$$\mathbf{v} = \frac{\mathbf{L}}{t}.$$

When we put these into (2) the time cancels out and we get (1).

If there is both an electric and a magnetic field the total force is the sum of the electric force and the magnetic force:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}.$$

This is called the Lorentz Force Law.

6 Proton in a Magnetic Field

Suppose we have a constant magnetic field pointed out of the paper,towards you. A proton moving to the left will experience a force a force towards of the top the page. As it changes its direction the force will change also, always pointing perpendicular to its velocity. The proton does not gain any kinetic energy: no work is done as the force is perpendicular to velocity. So its speed is a constant.

If a particle moves with constant speed in a plane, but its acceleration is everywhere perpendicular to its velocity, it is moving in a circle. The acceleration is pointed towards the center of the circle and has magnitude

$$a = \frac{v^2}{r}.$$

Putting mass times acceleration equal to the force

$$m\frac{v^2}{r} = qvB.$$

Thus the radius of the circle is

$$r = \frac{mv}{qB}.$$

Remember that that the angular velocity ω (the rate at which the angle changes) of a particle in circular motion is given by

$$v = r\omega.$$

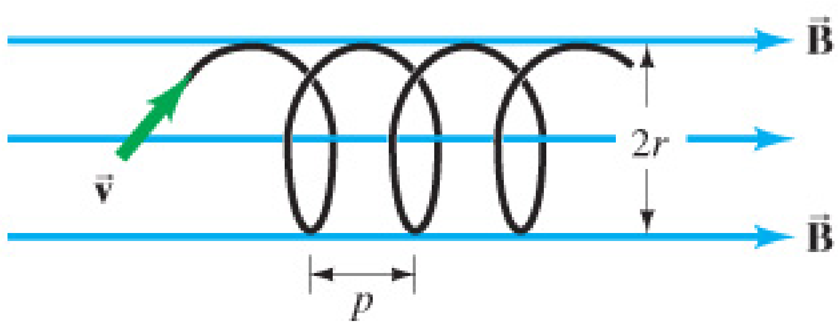
Thus we get

$$\omega = \frac{qB}{m}.$$

Thus the time it takes to make one complete revolution (turn through 2π) is

$$T = 2\pi\frac{m}{qB}$$

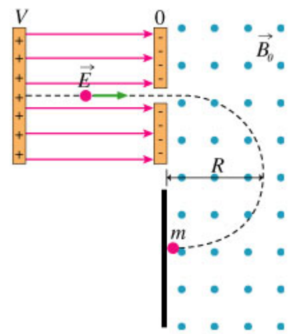
Note that this doesn't depend on the initial velocity.



If the initial velocity has a comonent along the magnetic field, that component will remain constant: there is no force in that direction. So the particle will move along a helix: each time it makes a circle in the plane normal to the field, it moves along the field some constant distance p. If $v_{\|}$ is the component of normal to the magnetic field and $v_{\|}$that parallel to the magnetic field,

$$p = Tv_{\|} = 2\pi\frac{mv_{\|}}{qB}.$$

7 Mass Spectrometer



Often chemists want to identify the various compounds a substance is made of. Every molecule as some mass. If we strip an electron (or add one) we can give it an electric charge. Then accelerate this ion through a known potential difference V. Now it has a kinetic energy

$$\frac{1}{2}mv^2 = qV.$$

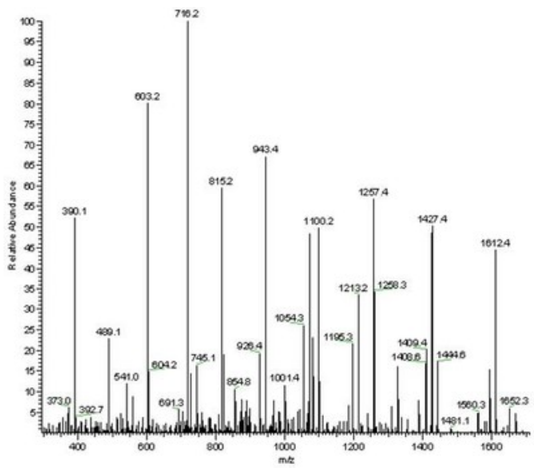
Its electric charge is $q = Ze$ where Z is the number of electrons stripped away to turn the molecule into an ion.

Now pass it through a constant magnetic field, entering the field in the plane normal to the field. The ion will move in a circular arc. Catch it on a detector after it has turned through some known angle (for example a semi-circle).

We can determine the radius R of the circular arc from the point where it hits the screen. Using this, we can figure out the ratio $\frac{m}{Z}$ of the ion.

Knowing $\frac{m}{Z}$, we can usually figure out what the molecule is. They have all been measured and tabulated. In practice the arc is smaller than a semi-circle. Also in the first stage there is another magnetic field to select ions of some known velocity. But the basic idea is the same.

By counting how many molecules hits a given spot on the detector we can even find out the proportion of each molecule in the substance.



8 Torque on a Coil

Consider a rectangular coil of wire of sides a and b. There is a uniform magnetic field B pointed along the sides of length b. The wire carries a current I. There is no force on the sides parallel to the magnetic field. Each of the other two sides experience a force equal in magnitude to $F = IaB$. The total force is zero, as the current is pointed in opposite directions.

But there will be a torque on the coil: it wants to turn so that both sides are perpendicular to the magnetic field. This torque has magnitude

$$\tau = Fb$$

because each side contributes $F\frac{b}{2}$: the force times the distance to the mid-point.In other words

$$\tau = IabB.$$

We can get a larger torque by winding the wire N times around the same path:

$$\tau = NIabB.$$

In practice there are coils that wind around several hundred times.The quantity $NIab$ (current times the area times the numbder of windings) is called the *magnetic moment* of the coil. It turns out that the shape of the wire doesn't matter: as long it is a curve that lies in some plane enclosing an are A, the torque is

$$\tau = \mu B$$

where the magnetic moment is

$$\mu = NIA.$$

Magnetic moment can be thought of as a vector: it has a direction perpendicular to the plane in which the coil lies. The torque also is a vector: the coil will want to turn in the direction of the torque. The general formula which gives the direction as well as magnitude of the torque is

$$\tau = \mu \times \mathbf{B}.$$

9 The Magnetic Moment of a Bar Magnet

A bar magnet also has a magnetic moment. It is a vector pointed from the North pole of the magnetic to its South pole. If a bar magnet is placed in a uniform magnetic field it will experience a torque: the force on each pole is equal but opposite in direction. The torque on the bar magnet (also called a dipole) is given by its magnetic moment times the magnetic field:

$$\tau = \mu \times \mathbf{B}$$

This is very similar to the formula for the torque on an electric dipole

$$\tau = \mathbf{p} \times \mathbf{E}.$$

Recall that an electric dipole is a pair of charges equal in magnitude q but opposite in sign separated by some distance $\mathbf{a}$. The electric dipole moment is

$$\mathbf{p} = q\mathbf{a}.$$

It is directed from the positive charge to the negative charge.

So a bar magnet is a lot like an electric dipole: the North pole is like the positive charge and the South pole is like a negative charge. But the big difference is that *you cannot separate a bar magnetic into an isolated North pole and an isolated South pole.*

Believe me, physicists have been trying to create 'magnetic monopoles' (isolated North or South poles) for many years. No one has succeded yet. As fas as we know, it is just impossible. If you cut a magnetic dipole, you will just get two magnetic dipoles. But it is possible to separate the positive and negative charges that make up an electric dipole.