Statics Review

charge density $\rho(\vec{r})$

$$\int_{\text{volume } V} d^3r\, \rho(\vec{r}) = Q \quad \text{total charge inside volume } V$$

current density $\vec{J}(\vec{r})$

$$\int_{\text{surface } S} d\vec{a} \cdot \vec{J}(\vec{r}) = I \quad \text{total current (charge per unit time) flowing through surface } S$$

local charge conservation

$$\frac{\partial \rho(\vec{r})}{\partial t} + \vec{\nabla} \cdot \vec{J}(\vec{r}) = 0$$

in electrostatics, $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \vec{\nabla} \cdot \vec{J}(\vec{r}) = 0$ is key condition for magnetostatics

– also $\frac{\partial \vec{J}}{\partial t} = 0$

Maxwell's Equations

electrostatics | **magnetostatics**

integral form:

Gauss $\oint_{\text{surface } S} \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\varepsilon_0}$ | $\oint_S \vec{B} \cdot d\vec{a} = 0$

$\oint_{\text{curve } C} \vec{E} \cdot d\vec{l} = 0$ | Ampere $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

differential form:

$\vec{\nabla} \cdot \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\varepsilon_0}$ | $\vec{\nabla} \cdot \vec{B}(\vec{r}) = 0$

$\vec{\nabla} \times \vec{E}(\vec{r}) = 0$ | $\vec{\nabla} \times \vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r})$

Potentials

electrostatic potential $V(\vec{r})$

$$\vec{E} = -\vec{\nabla} V$$

magnetostatic vector potential $\vec{A}(\vec{r})$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

can also choose $\vec{A}$ to satisfy the additional condition $\vec{\nabla} \cdot \vec{A} = 0$ see Griffiths see ~~[illegible]~~ 5.4.1

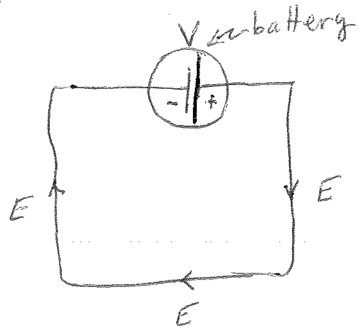
~~Read 6.4.2 ferromag~~

Magnetic Induction + Faradays Law Read 7.1.1

electromotive force = "emf" = $\mathcal{E}$ is the work done, per unit charge, to move a current around a closed loop

$$\mathcal{E} = \oint \vec{f}\cdot d\vec{l} \quad \text{where } \vec{f} \text{ is the force per unit charge.}$$

ex: For a simple wire loop of length L, connected to a battery of voltage V, there is an electric field in the wire $E = \frac{V}{L}$, pointing tangential to the wire



force on electrons is $\vec{F} = -e\vec{E}$

force per unit charge is $\vec{f} = \frac{\vec{F}}{-e} = \vec{E}$

work done per electron is

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = EL = V$$

in doing integral, we left out the section containing the battery. If we included the section containing the battery, then in this portion $\int \vec{E}\cdot d\vec{l} = -V$, since $\oint E\cdot dl$ around a closed loop is zero as $\vec{\nabla}\times\vec{E} = 0$; but we would also have to include the chemical force supplied by the battery, and this $\int \vec{f}_{battery}\cdot d\vec{l} = V$ - this just defines the voltage supplied by the battery. So the electric and chemical force contributions cancell in the battery segment, and the net emf is $\mathcal{E} = V$ as computed above.

In definition of $\mathcal{E}$, direction one goes around loop is arbitrary.
If $\mathcal{E}$ comes out > 0, then current flows in direction of integration
If $\mathcal{E}$ comes out < 0, then current flows in opposite direction to integration

Induction

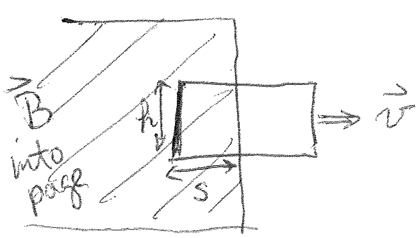
A changing magnetic flux through a loop, induces an emf around the loop

integrate over area bounded by loop

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}} \quad \text{where } \Phi = \int \vec{B}\cdot d\vec{a} \quad \text{flux of } \vec{B} \text{ through the loop}$$

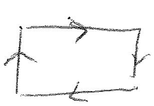
signs: if go ◯ in computing $\mathcal{E}$, then take outward normal $d\vec{a} = \hat{n}\,da$ in computing Φ.
If go ◯ in computing $\mathcal{E}$, then take inward normal in computing Φ.
Always use right hand rule.

proof in a simple example



pull wire loop as shown

as pull loop, charges in left side experience Lorentz force $\vec{F} = q\,\vec{v}\times\vec{B}$ causing current to flow upwards along wire (Lorentz forces on top + bottom segments are $\perp$ to wire, so they dont drive any current)



integrate clockwise

$$\mathcal{E} = \oint \vec{f}_{mag}\cdot d\vec{l} = \oint (\vec{v}\times\vec{B})\cdot d\vec{l} = vBh$$

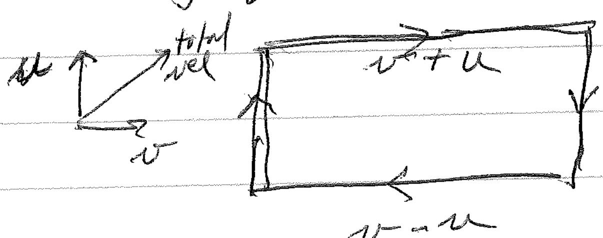
flux through loop is $\Phi = B h s$

$$\frac{d\Phi}{dt} = B h \frac{ds}{dt} = -B h v$$

sign negative as s is decreasing

$$\Rightarrow \mathcal{E} = -\frac{d\Phi}{dt}$$

When we worked out the emf around the loop above we considered the velocity $\vec{v}$ of the loop, but we ignored the velocity of the charges going around the loop. Lets now add that, say the speed ~~velocity~~ of charges traveling down the wire is u.



$\Rightarrow$ loop pulled to right with $\vec{v}$

The Lorentz force on the horizontal segments of the wire is still in the vertical direction $\Rightarrow$ does not contribute to the emf.

The Lorentz force on the left vertical segment is

$$q(\vec{v}+\vec{u}) \times \vec{B} = q\vec{v} \times \vec{B} + q\vec{u} \times \vec{B}$$

oriented parallel to the wire — contributes to emf

oriented opposite to direction $\vec{v}$, orthogonal to wire does not contribute to emf