

## Electric field

from Ampere,  $\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{\nabla} \times \vec{B}$  since  $\vec{j} = 0$   
 far from source

$$\Rightarrow \vec{E}_{E1} = \frac{i}{\omega \mu_0 \epsilon_0} \vec{\nabla} \times \vec{B}_{E1} \quad \text{since } \frac{\partial \vec{E}}{\partial t} = -i\omega \vec{E}$$

$$= \frac{-i}{\omega \mu_0 \epsilon_0} \frac{c \mu_0}{4\pi} k^2 \vec{\nabla} \times \left( \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr}\right) \vec{p} \times \hat{r} \right)$$

$$\omega = ck$$

to evaluate the  $\vec{\nabla} \times (\ )$  term, we  $\vec{\nabla} \times (f\vec{g}) = f\vec{\nabla} \times \vec{g} + \vec{\nabla} f \times \vec{g}$   
 with  $f = e^{ikr}$ ,  $\vec{g} = \frac{1}{r} \left(1 + \frac{i}{kr}\right) \vec{p} \times \hat{r}$

$$\text{then } \vec{\nabla} \times (\ ) = \left( \vec{\nabla} e^{ikr} \right) \times \left( \frac{1}{r} \left(1 + \frac{i}{kr}\right) \vec{p} \times \hat{r} \right)$$

$$+ e^{ikr} \vec{\nabla} \times \left( \frac{1}{r} \left(1 + \frac{i}{kr}\right) \vec{p} \times \hat{r} \right)$$

But in the radiation zone we can ignore the second term, since

$$\vec{\nabla} \times \left( \frac{1}{r} \left(1 + \frac{i}{kr}\right) \vec{p} \times \hat{r} \right) \sim \frac{1}{r^2}$$

we see this by noting that

$$\vec{\nabla} \frac{1}{r} \sim \frac{1}{r^2}, \quad \vec{\nabla} \frac{1}{r^2} \sim \frac{1}{r^3}$$

$$\vec{\nabla} \hat{r} \sim \frac{\partial}{\partial x} \frac{\vec{r}}{|\vec{r}|} = \frac{\hat{x}}{|\vec{r}|} - \frac{\vec{r}}{|\vec{r}|^2} \frac{x}{|\vec{r}|} = \frac{\hat{x}}{|\vec{r}|} - \frac{x\hat{r}}{|\vec{r}|^2} \sim 0 \left(\frac{1}{r}\right)$$

So keep only 1st term in Radiation Zone and we get

$$\vec{E}_{E1} = \frac{-i}{\omega \mu_0 \epsilon_0} \frac{c \mu_0}{4\pi} k^2 \left( i k e^{ikr} \hat{r} \right) \times \left( \frac{1}{r} \left(1 + \frac{i}{kr}\right) \vec{p} \times \hat{r} \right)$$

use  $\omega = ck$

$$= \vec{\nabla} e^{ikr}$$

$\frac{1}{r}$  ignore in RZ

$$\vec{E}_{E1} = \frac{k^2}{4\pi \epsilon_0} \frac{e^{ikr}}{r} \hat{r} \times (\vec{p} \times \hat{r})$$

## electric field

from Ampere,  $\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B}$ , since  $\vec{j} = 0$  for free space

$$\Rightarrow \vec{E}_{E1} = \frac{i}{\omega \mu_0 \epsilon_0} \nabla \times \vec{B}_{E1} \quad \text{since } \frac{\partial \vec{E}}{\partial t} = -i\omega \vec{E}$$

$$= \frac{-i}{\omega \mu_0 \epsilon_0} \frac{c \mu_0}{4\pi} k^2 \nabla \times \left( \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr}\right) \vec{p} \times \hat{r} \right)$$

$\omega = ck$

to evaluate  $\nabla \times ( )$ , use  $\nabla \times (f\vec{g}) = f \nabla \times \vec{g} + \nabla f \times \vec{g}$   
with  $f = \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr}\right)$  and  $\vec{g} = \vec{p} \times \hat{r}$

$$\nabla \times ( ) = \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr}\right) \nabla \times (\vec{p} \times \hat{r}) + \nabla \left( \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr}\right) \right) \times (\vec{p} \times \hat{r})$$

evaluate second term:  $\nabla \left( \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr}\right) \right) = \frac{\partial}{\partial r} \left( \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr}\right) \right) \hat{r}$  in spherical coords

$$= e^{ikr} \left[ ik \left( \frac{1}{r} + \frac{i}{kr^2} \right) - \frac{1}{r^2} - \frac{2i}{kr^3} \right] \hat{r}$$

$$= \frac{e^{ikr}}{r} \left[ ik - \frac{2}{r} - \frac{2i}{kr^2} \right] \hat{r}$$

evaluate first term:

$$\nabla \times (\vec{p} \times \hat{r}) = \vec{p} (\nabla \cdot \hat{r}) - (\vec{p} \cdot \nabla) \hat{r}$$

$$\text{where } \nabla \cdot \hat{r} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2) = \frac{2}{r}$$

cover Griffiths evaluating in spherical coordinates

$$\text{and } (\vec{p} \cdot \nabla) \hat{r} = \sum_k p_k \frac{\partial \hat{r}}{\partial r_k}$$

unit vector in  $k$  direction

$$\text{where } \frac{\partial \hat{r}}{\partial r_k} = \frac{\partial}{\partial r_k} \left( \frac{\vec{r}}{r} \right) = \vec{r} \left( -\frac{1}{r^2} \frac{\partial r}{\partial r_k} \right) + \frac{\hat{e}_k}{r}$$

$$= \vec{r} \left( -\frac{1}{r^2} \frac{r_k}{r} \right) + \frac{\hat{e}_k}{r} \quad \text{as } \frac{\partial r}{\partial r_k} = \frac{r_k}{r}$$

$$\begin{aligned}
 \text{So } \vec{\nabla} \times (\vec{p} \times \hat{r}) &= \frac{2\vec{p}}{r} - \sum_k p_k \left( -\frac{\vec{r}}{r^3} r_k + \frac{\hat{e}_k}{r} \right) \\
 &= \frac{2\vec{p}}{r} + \frac{\vec{r}}{r^3} \vec{p} \cdot \vec{r} - \frac{\vec{p}}{r} \\
 &= \frac{\vec{p}}{r} + \hat{r} (\vec{p} \cdot \hat{r}) \quad \text{using } \hat{r} = \frac{\vec{r}}{r}
 \end{aligned}$$

putting all the pieces together

$$\begin{aligned}
 \vec{E}_{E1} &= \frac{-ik}{4\pi\epsilon_0} \frac{e^{ikr}}{r} \left[ (1 + \frac{i}{kr}) \frac{\vec{p} + \hat{r}(\vec{p} \cdot \hat{r})}{r} \right. \\
 &\quad \left. + (ik - \frac{2}{r} - \frac{2i}{kr^2}) \frac{\hat{r} \times (\vec{p} \times \hat{r})}{\vec{p} - \hat{r}(\vec{p} \cdot \hat{r})} \right]
 \end{aligned}$$

order by powers of  $\frac{1}{r}$

$$\begin{aligned}
 &= \frac{-ik}{4\pi\epsilon_0} \frac{e^{ikr}}{r} \left[ ik (\vec{p} - \hat{r}(\vec{p} \cdot \hat{r})) + \frac{1}{r} (1 + \frac{i}{kr}) (\vec{p} + \hat{r}(\vec{p} \cdot \hat{r})) \right. \\
 &\quad \left. - \frac{2}{r} (1 + \frac{i}{kr}) (\vec{p} - \hat{r}(\vec{p} \cdot \hat{r})) \right] \\
 &= \frac{k^2}{4\pi\epsilon_0} \frac{e^{ikr}}{r} \left[ \vec{p} - \hat{r}(\vec{p} \cdot \hat{r}) - \frac{i}{kr} (1 + \frac{i}{kr}) (\vec{p} + \hat{r}(\vec{p} \cdot \hat{r})) \right. \\
 &\quad \left. - 2\vec{p} + 2\hat{r}(\vec{p} \cdot \hat{r}) \right]
 \end{aligned}$$

$$\boxed{\vec{E}_{E1} = \frac{k^2}{4\pi\epsilon_0} \frac{e^{ikr}}{r} \left[ \vec{p} - \hat{r}(\vec{p} \cdot \hat{r}) - \frac{i}{kr} (1 + \frac{i}{kr}) (3\hat{r}(\vec{p} \cdot \hat{r}) - \vec{p}) \right]}$$

radiation zone approx  $kr \gg 1$  keep only terms of order  $\frac{1}{r}$

$$\boxed{\vec{E}_{E1} = \frac{k^2}{4\pi\epsilon_0} \frac{e^{ikr}}{r} \left[ \underbrace{\vec{p} - \hat{r}(\vec{p} \cdot \hat{r})}_{\hat{r} \times (\vec{p} \times \hat{r})} \right]}$$

~~radiation zone limit~~ radiation zone limit

$$\vec{E}_{E1} = \frac{k^2}{4\pi\epsilon_0} \frac{e^{ikr}}{r} \hat{r} \times (\vec{p} \times \hat{r})$$

$$\vec{B}_{E1} = -\frac{c\mu_0}{4\pi} k^2 \frac{e^{ikr}}{r} \vec{p} \times \hat{r}$$

radiation zone fields  
in electric dipole approx

$\vec{E}$  and  $\vec{B}$  are outwards traveling spherical waves  
 $\sim \frac{e^{ikr}}{r}$

$$\frac{|\vec{B}_{E1}|}{|\vec{E}_{E1}|} = \frac{c\mu_0}{4\pi} \cdot 4\pi\epsilon_0 = c\mu_0\epsilon_0 \stackrel{\text{using } \mu_0\epsilon_0 = 1/c^2}{=} \frac{c}{c^2} = \frac{1}{c}$$

just as for plane waves in vacuum

if choose coordinates so that  $\vec{p}$  is along  $\hat{z}$  axis

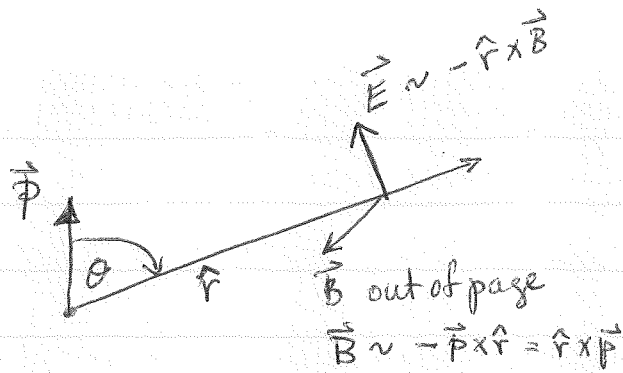


$$\begin{aligned} \vec{p} \times \hat{r} &= p \sin\theta \hat{\phi} \\ \hat{r} \times (\vec{p} \times \hat{r}) &= p \hat{r} \times \hat{\phi} = -\hat{\theta} \sin\theta p \end{aligned}$$

$$\vec{E}_{E1} = -\frac{k^2 p}{4\pi\epsilon_0} \frac{e^{ikr}}{r} \sin\theta \hat{\theta}$$

$$\vec{B}_{E1} = -\frac{c\mu_0}{4\pi} k^2 p \frac{e^{ikr}}{r} \sin\theta \hat{\phi}$$

$\hat{\theta}$  &  $\hat{\phi}$  are  
spherical coord  
basis vectors



$\vec{E}$  is in plane containing  $\vec{p}$  and  $\hat{r}$   
 $\vec{B}$  is  $\perp$  plane containing  $\vec{p}$  and  $\hat{r}$

$\vec{E}_{E1}$  and  $\vec{B}_{E1}$  are orthogonal, as in plane wave,  
 and both are orthogonal to direction of propagation  $\hat{r}$

$\Rightarrow$  oscillating source emits spherical electromagnetic waves.

What is power emitted?

Poynting vector:  $\vec{S}_{E1}(\vec{r}, t) = \frac{1}{\mu_0} \text{Re}[\vec{E}_{E1}(\vec{r}, t)] \times \text{Re}[\vec{B}_{E1}(\vec{r}, t)]$

$$\text{Re}[\vec{E}_{E1}(\vec{r}, t)] = \text{Re}\left[ \frac{k^2}{4\pi\epsilon_0} \frac{e^{i(kr - \omega t)}}{r} \hat{r} \times (\vec{p} \times \hat{r}) \right]$$

$$= \frac{k^2}{4\pi\epsilon_0} \frac{\cos(kr - \omega t)}{r} \hat{r} \times (\vec{p} \times \hat{r})$$

\*\*  
 assuming  
 $\vec{p}$  is real  
 \*\*

$$\text{Re}[\vec{B}_{E1}(\vec{r}, t)] = -\frac{c\mu_0}{4\pi} k^2 \frac{\cos(kr - \omega t)}{r} (\vec{p} \times \hat{r})$$

$$\vec{S}_{E1} = \frac{1}{\mu_0} \frac{k^2}{4\pi\epsilon_0} \frac{c\mu_0 k^2}{4\pi} \frac{\cos^2(kr - \omega t)}{r^2} [\hat{r} \times (\vec{p} \times \hat{r})] \times [\vec{p} \times \hat{r}]$$

$$= -\frac{ck^4}{(4\pi)^2 \epsilon_0} \frac{\cos^2(kr - \omega t)}{r^2} \underbrace{[\vec{p} - \hat{r}(\hat{r} \cdot \vec{p})] \times [\vec{p} \times \hat{r}]}$$


$$(\vec{p}(\vec{p} \cdot \hat{r}) - \hat{r}p^2) - (\hat{r} \cdot \vec{p})(\vec{p} - \hat{r}(\vec{p} \cdot \hat{r}))$$

$$= \hat{r}((\vec{p} \cdot \hat{r}) - p^2)$$

$$\vec{S}_{EI} = \frac{ck^4}{(4\pi)^2 \epsilon_0} \frac{\cos^2(kr - \omega t)}{r^2} [p^2 - (\vec{p} \cdot \hat{r})^2] \hat{r}$$

in radial direction

if define  $\theta$  as angle between  $\vec{p}$  and  $\hat{r}$



then  $p^2 - (\vec{p} \cdot \hat{r})^2 = p^2(1 - \cos^2\theta) = p^2 \sin^2\theta$

$$\vec{S}_{EI} = \frac{ck^4 p^2}{(4\pi)^2 \epsilon_0} \frac{\cos^2(kr - \omega t)}{r^2} \sin^2\theta \hat{r}$$

points radially outward!

average over one period of oscillation  $\langle \cos^2(kr - \omega t) \rangle = \frac{1}{2}$

$$\langle \vec{S}_{EI} \rangle = \frac{ck^4 p^2}{2(4\pi)^2 \epsilon_0} \frac{\sin^2\theta}{r^2} \hat{r}$$

note, the  $\frac{1}{2}$  is important for energy conservation.

If we integrate  $\langle \vec{S}_{EI} \rangle$  over the surface of a sphere of radius  $r$ , the result is indep of  $r$ .

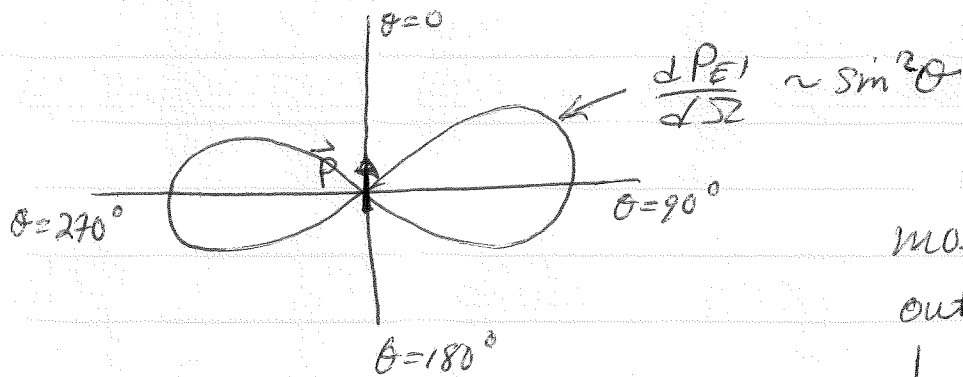
= power  
Average energy flux flowing through an element of area at spherical angles  $\theta, \phi$  is

$$dP_{EI} = \hat{r} \cdot \langle \vec{S}_{EI} \rangle \underbrace{r^2 \sin\theta d\theta d\phi}_{\text{area of surface element} = r^2 d\Omega}$$

$d\Omega = \sin\theta d\theta d\phi$  differential solid angle

$$= \hat{r} \cdot \langle \vec{S}_{EI} \rangle r^2 d\Omega$$

$$\frac{dP_{E1}}{d\Omega} = \hat{r} \cdot \langle \vec{S}_{E1} \rangle r^2 = \frac{ck^4 p^2}{2(4\pi)^2 \epsilon_0} \sin^2 \theta \sim \omega^4 \sin^2 \theta$$



most of power is directed outwards into the plane  $\perp$  to  $\vec{p}$ , i.e. at angles  $\theta$  peaked about  $90^\circ$

For energy conservation to hold, it must be true that all the higher order terms, that go as higher powers of  $\frac{1}{r}$  (i.e.  $\frac{1}{r^2}$ ,  $\frac{1}{r^3}$ , etc ...), must vanish when compute the time averaged energy flux <sup>integrated over surface of sphere</sup> otherwise energy would be disappearing as the wave propagated outwards.

Total power radiated is

$$P_{E1} = \int \frac{dP_{E1}}{d\Omega} d\Omega = \frac{ck^4 p^2}{2(4\pi)^2 \epsilon_0} \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \sin^2 \theta$$

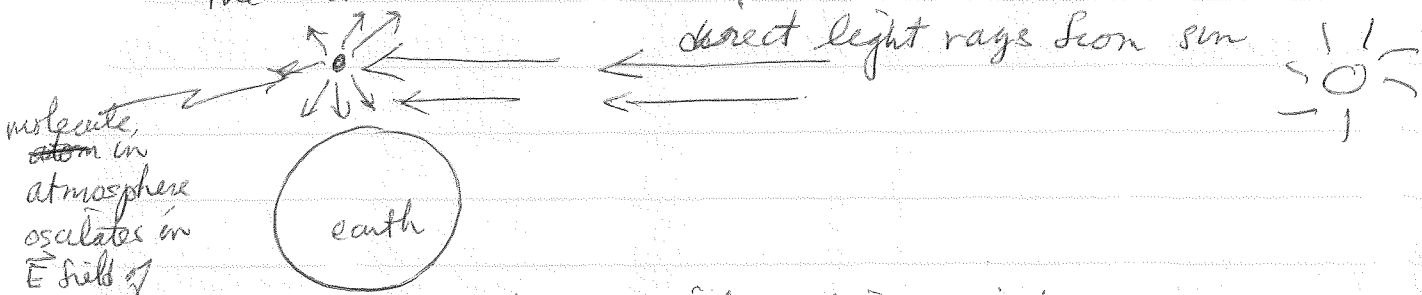
$$= \frac{ck^4 p^2}{32\pi^2 \epsilon_0} 2\pi \int_0^\pi d\theta \sin \theta (1 - \cos^2 \theta)$$

$$\left[ -\cos \theta + \frac{\cos^3 \theta}{3} \right]_0^\pi = \frac{4}{3}$$

$$P_{E1} = \frac{ck^4 p^2}{4\pi \epsilon_0 \cdot 3} = \boxed{\frac{p^2 \omega^4}{4\pi \epsilon_0 3c^3} = P_{E1} \sim \omega^4}$$

## Why the sky is blue - Lord Rayleigh

When look at sky, you are seeing the indirect light of the sun, which is the light emitted by the atoms and ~~noted~~ molecules of the atmosphere as they oscillate + so radiate, due to the electric field of the direct light from the sun



direct rays, and then emits radiated light with power  
(can view this as a scattering of the direct rays)

$$P \sim \omega^4 p^2$$

$p$  is dipole moment of ~~atom~~ molecule in atmosphere  $p = \alpha E$

polarizability

↑ electric field of direct rays

$$\alpha \sim \frac{e^2}{m} \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

for molecules in atmosphere,  $N_2$ , etc,  $\omega_0$  is typically a freq higher than the visible spectrum. Therefore for light in visible spectrum,  $\alpha \sim \frac{e^2}{m\omega_0^2}$  indep of  $\omega$

Power emitted  $\sim \omega^4$  largest at higher freq.

Since light from sun is "white light" it has components of all freqs. ~~For~~ From the above, we see that this indirect scattered light is most scattered at the higher freqs, due to the  $\omega^4$  dependence of



Scattered power in electric dipole approx  
⇒ indirect light is strongest in the blue (large  $\omega$ )  
part of the visible spectrum, ⇒ sky is blue!

When we look at sunrise or sunset however,  
we are looking at the direct rays of the sun.  
Since these rays are scattered most in the blue,  
the direct rays are strongest in the red (small  $\omega$ )  
part of the spectrum → sunset & sunrise are red!

Note: when we wrote

$$\text{Re} [\vec{E}_{E1}(\vec{r}, \omega) e^{-i\omega t}] = \frac{k^2}{4\pi\epsilon_0} \frac{\cos(kr - \omega t)}{r} \hat{r} \times (\vec{p} \times \hat{r})$$

we implicitly assumed that the amplitude of the oscillating electric dipole moment  $\vec{p}(\omega)$  was a real vector. But that is not necessarily always the case!

For  $\vec{p}(\omega) \equiv \vec{p}_1$  real, the time dependent dipole moment is

$$\vec{p}(t) = \text{Re} [\vec{p}_1 e^{-i\omega t}] = \vec{p}_1 \cos \omega t$$

points always in same direction with oscillating magnitude.

But suppose  $\vec{p}(\omega) = \vec{p}_1 + i\vec{p}_2$ . Then

$$\begin{aligned} \vec{p}(t) &= \text{Re} [(\vec{p}_1 + i\vec{p}_2) e^{-i\omega t}] \\ &= \vec{p}_1 \cos \omega t + \vec{p}_2 \sin \omega t \end{aligned}$$

Now direction of  $\vec{p}(t)$  is rotating!

If  $\vec{p}_1 \perp \vec{p}_2$  then the tip of  $\vec{p}(t)$  sweeps out an ellipse! An example of such a  $\vec{p}$  would be a charge moving in an elliptical orbit

So if  $\vec{p}(\omega)$  is complex we need to be more careful in our calculation of  $\vec{S}$

# Magnetic Dipole Radiation - in the Radiation Zone Approx

$$\vec{A}_M = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} ik \hat{r} \times \vec{m}$$

$$\vec{B}_M = \vec{\nabla} \times \vec{A}_M = \frac{\mu_0}{4\pi} ik \vec{\nabla} \times \left( e^{ikr} \frac{\hat{r} \times \vec{m}}{r} \right)$$

Exactly the same form as when we computed  $\vec{E}_E$  from  $\vec{B}_E$  except  $\vec{p} \rightarrow \vec{m}$

$$\text{use } \vec{\nabla} \times (f\vec{g}) = \vec{\nabla} f \times \vec{g} + f \vec{\nabla} \times \vec{g}$$

$$\vec{B}_M = \frac{\mu_0}{4\pi} ik \left[ \underbrace{(\vec{\nabla} e^{ikr}) \times \left( \frac{\hat{r} \times \vec{m}}{r} \right)}_{ike^{ikr} \hat{r} \times \left( \frac{\hat{r} \times \vec{m}}{r} \right)} + e^{ikr} \underbrace{\vec{\nabla} \times \left( \frac{\hat{r} \times \vec{m}}{r} \right)}_{\sim O\left(\frac{1}{r^2}\right) \text{ so ignore in RZ approx}} \right]$$

$$\vec{B}_M = \frac{-\mu_0}{4\pi} k^2 \frac{e^{ikr}}{r} \hat{r} \times (\hat{r} \times \vec{m})$$

$$\vec{E}_M = \frac{i}{\omega \mu_0 \epsilon_0} \vec{\nabla} \times \vec{B}_M \quad \text{from Amperes law with } \vec{j} = 0$$

$$= \frac{-i \mu_0 k^2}{4\pi \omega \mu_0 \epsilon_0} \vec{\nabla} \times \left( \frac{e^{ikr}}{r} \hat{r} \times (\hat{r} \times \vec{m}) \right)$$

$$= \frac{-ik^2}{4\pi \omega \epsilon_0} \left[ \underbrace{(\vec{\nabla} e^{ikr}) \times \left( \frac{\hat{r} \times (\hat{r} \times \vec{m})}{r} \right)}_{ike^{ikr} \hat{r} \times (\hat{r} \times \vec{m})} + e^{ikr} \underbrace{\vec{\nabla} \times \left( \frac{\hat{r} \times (\hat{r} \times \vec{m})}{r} \right)}_{\sim O\left(\frac{1}{r^2}\right) \text{ so ignore in RZ approx}} \right]$$

$$= \frac{k^3}{4\pi \omega \epsilon_0} \frac{e^{ikr}}{r} \hat{r} \times (\hat{r} \times (\hat{r} \times \vec{m})) \quad \text{use } \omega = ck$$

use triple product rule

$$\hat{r} \times (\hat{r} \times (\hat{r} \times \vec{m})) = \hat{r} (\hat{r} \cdot (\hat{r} \times \vec{m})) - (\hat{r} \times \vec{m}) (\hat{r} \cdot \hat{r})$$

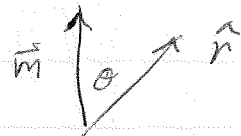
$$\vec{E}_M = \frac{-k^2}{4\pi \epsilon_0 c} \frac{e^{ikr}}{r} \hat{r} \times \vec{m}$$

$$= 0 - \hat{r} \times \vec{m}$$

$$\vec{E}_{MI} = \frac{k^2}{4\pi\epsilon_0 c} \frac{e^{ikr}}{r} \left[ -\hat{r} \times \vec{m} \right]$$

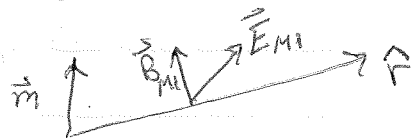
$$\vec{E}_{MI} = \frac{k^2}{4\pi\epsilon_0 c} \frac{e^{ikr}}{r} \left[ \vec{m} \times \hat{r} \right]$$

$$\vec{B}_{MI} = \frac{\mu_0 k^2}{4\pi} \frac{e^{ikr}}{r} \left[ \hat{r} \times (\vec{m} \times \hat{r}) \right]$$



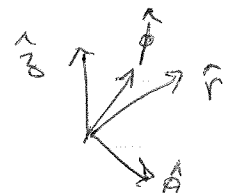
for  $\vec{m} = m \hat{\phi}$ ,  $\vec{m} \times \hat{r} = m \sin\theta \hat{\phi}$   
 $\hat{r} \times (\vec{m} \times \hat{r}) = m \sin\theta (-\hat{\theta})$

$$\left. \begin{aligned} \vec{E}_{MI} &= \frac{k^2 m}{4\pi\epsilon_0 c} \frac{e^{ikr}}{r} \sin\theta \hat{\phi} \\ \vec{B}_{MI} &= -\frac{\mu_0 k^2 m}{4\pi} \frac{e^{ikr}}{r} \sin\theta \hat{\theta} \end{aligned} \right\} \begin{array}{l} \text{Note similarity with } \vec{E}_{E1} \text{ and } \vec{B}_{E1} \\ \hat{\phi} \rightarrow \frac{\vec{m}}{c}, \vec{E} + \vec{B} \text{ rotated by } 90^\circ \end{array}$$



Poynting vector

$$\vec{S}_{MI} = \frac{1}{\mu_0} \vec{E}_{MI} \times \vec{B}_{MI} = \frac{1}{\mu_0} \left( \frac{k^2 m}{4\pi\epsilon_0 c} \frac{e^{ikr}}{r} \sin\theta \hat{\phi} \right) \times \left( -\frac{\mu_0 k^2 m}{4\pi} \frac{e^{ikr}}{r} \sin\theta \hat{\theta} \right)$$



$$\vec{S}_{MI} = \frac{1}{\mu_0} \text{Re} \left\{ \vec{E}_{MI} e^{-i\omega t} \right\} \times \text{Re} \left\{ \vec{B}_{MI} e^{-i\omega t} \right\}$$

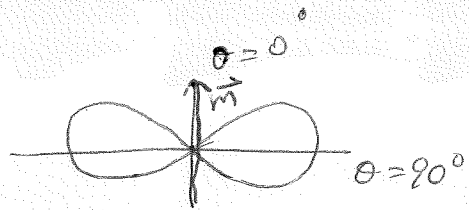
$$= \frac{1}{\mu_0} \left( \frac{k^2 m}{4\pi\epsilon_0 c} \right) \left( -\frac{\mu_0 k^2 m}{4\pi} \right) \frac{\sin^2\theta}{r^2} \cos^2(kr - \omega t) \underbrace{\hat{\phi} \times \hat{\theta}}_{-\hat{r}}$$

$$= \frac{k^4 m^2}{4\pi\epsilon_0 4\pi c} \frac{\sin^2\theta}{r^2} \cos^2(kr - \omega t) \hat{r}$$

time average

$$\langle \vec{S}_{MI} \rangle = \frac{k^4 m^2}{32\pi^2 \epsilon_0 c} \frac{\sin^2\theta}{r^2} \hat{r}$$

power cross section



$$\frac{dP_M}{d\Omega} = \hat{r} \cdot \langle \vec{S}_M \rangle r^2 = \frac{k^4 m^2 \sin^2 \theta}{2(4\pi)^2 \epsilon_0 c} = \frac{dP_M}{d\Omega}$$

same form as  $\frac{dP_{E1}}{d\Omega}$  with  $p \rightarrow \frac{m}{c}$

total power

$$P_M = \int \frac{dP_M}{d\Omega} d\Omega = \frac{2\pi k^4 m^2}{2(4\pi)^2 \epsilon_0 c} \int_0^\pi \sin^3 \theta d\theta$$

$$k^4 = \frac{\omega^4}{c^4}$$

$$P_M = \frac{\omega^4 m^2}{4\pi \epsilon_0 3c^5}$$

compare to  $P_{E1} = \frac{\omega^4 p^2}{4\pi \epsilon_0 3c^3}$

$$\frac{P_M}{P_{E1}} = \left(\frac{m}{cp}\right)^2 \quad \text{as } m \sim v p \Rightarrow \frac{P_M}{P_{E1}} \sim \left(\frac{v}{c}\right)^2$$

electric quadrupole radiation - radiation zone approx

$$\vec{A}_{E2} = \frac{\mu_0}{4\pi r} e^{ikr} ik \left(\frac{i\omega}{6}\right) A \cdot \hat{r}$$

check

Fwd fields for homework!