PHY 415 **Midterm Exam** **Fall 2007**

1) [38 points]
A spherical shell of radius R carries a uniform surface charge density $+\sigma_0$ on the northern hemisphere, $0 < \theta < \pi/2$, and an equal but opposite uniform surface charge density $-\sigma_0$ on the southern hemisphere, $\pi/2 < \theta < \pi$. Calculating the coefficients in the Legendre expansion up to $\ell = 6$,

a) find the electrostatic potential outside the sphere

b) find the electrostatic potential inside the sphere

2) [38 points]
Consider a line charge density $\lambda(z)$ that is localized on the z axis from $z = -a$ to $z = +a$. By considering the monopole, dipole, and quadrapole moments of the charge distribution, find an approximation for the electrostatic potential $\phi(\mathbf{r})$ to *leading order only* in the multipole expansion, for each of the following three cases:

a) $\lambda(z) = \lambda_0 \cos(\pi z/2a)$

b) $\lambda(z) = \lambda_0 \sin(\pi z/a)$

c) $\lambda(z) = \lambda_0 \cos(\pi z/a)$

3) [24 points]
Consider a thin flat circular disk of radius R centered about the origin in the xy plane. The disk has a uniform surface charge density σ_0 and is rotating about the z axis with an angular velocity ω. For positions $\mathbf{r}$ far from the disk, $r >> R$, write an approximation for the magnetic field $\mathbf{B}(\mathbf{r})$.

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \tfrac{1}{2}(3x^2 - 1)$$

$$P_3(x) = \tfrac{1}{2}(5x^3 - 3x)$$

$$P_4(x) = \tfrac{1}{8}(35x^4 - 30x^2 + 3)$$

$$P_5(x) = \tfrac{1}{8}(63x^5 - 70x^3 + 15x)$$

$$P_6(x) = \tfrac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5)$$