PHY 415 **Endterm Exam** **Fall 2023**

This exam is closed book, closed notes. You may not consult with any other person or resource, except for the formulae sheet provided with this exam and your one page "cheat sheet." Please write clearly and use a dark pen or pencil. The better you explain your steps, the more likely you are to get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at.

Please write the academic honesty pledge, and sign your name, at the top of your work: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

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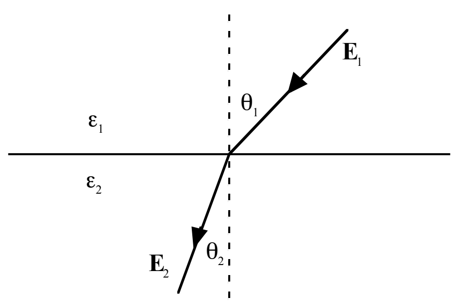
1) [30 points total] - This problem is a series of relatively short answer questions. The algebra is meant to be relatively simple if you know what you are doing! Each part is worth 10 points. The various parts are unrelated.

a) [10 pts] Consider a charge q located a distance r away from the center of a small neutral dielectric sphere of radius a and dielectric constant ϵ. Assume $a \ll r$. Consider the force between the charge and the dielectric sphere. How does it depend on the distance r? Is it attractive or repulsive? [Hint: to do this problem you should not need any lengthy calculations.]

b) [10 pts] Consider two semi-infinite dielectrics with real positive dielectric constants ϵ_1 and ϵ_2 that meet at a plane interface as in the figure below. If a *static* uniform electric field $\mathbf{E}_1$ is present in dielectric 1, making an angle θ_1 with respect to the normal to the interface, then what is the angle θ_2 of the static uniform static electric field $\mathbf{E}_2$ in dielectric 2? Assume that there is no free charge at the interface.



c) [10 pts] Consider a very long straight wire of length L and radius a carrying a uniform steady current I. If the wire has a uniform resistance per unit length, R/L, then there will be a voltage drop down the length of the wire, $V = IR$, and hence an electric field in the wire, $E = V/L$. Find the rate of electromagnetic energy flowing through the surface of the wire. (Assume that L is so long that you may ignore the effects at the ends of the wire.) Does energy flow into or out of the wire? You answer should look familiar. Give a physical explanation for your result.

(turn over for problems 2 and 3)

2) [35 points total]

a) [12 pts] Consider the propagation of a linearly polarized transverse plane electromagnetic wave traveling in the $\hat{\mathbf{z}}$ direction in an infinite dielectric material. In lecture we considered the behavior as a function of the wave frequency ω and found that there were three different regimes: transparent propagation, resonant absorption, and total reflection. Describe the distinguishing features of the complex dielectric function $\varepsilon(\omega)$ and the complex wavevector $\mathbf{k} = k\hat{\mathbf{z}}$, in each of these three regimes.

b) [8 pts] Consider the dielectric function $\varepsilon(\omega)$ of a conductor and explain qualitatively the main difference between this and the $\varepsilon(\omega)$ of a dielectric insulator. Give a clear physical example of a difference in behavior of transverse plane wave propagation between the conductor and the dielectric.

c) [15 pts] Consider a linearly polarized transverse plane electromagnetic wave traveling in the $\hat{\mathbf{z}}$ direction with frequency ω in the region of total reflection of either a dielectric or conductor. Compute the time averaged Poynting vector*

$$\langle \mathbf{S}(\mathbf{r},t) \rangle = \frac{c}{4\pi} \langle \mathbf{E}(\mathbf{r},t) \times \mathbf{H}(\mathbf{r},t) \rangle$$

and explain why your result is consistent with this being a region of total reflection. For this calculation you may assume that μ is a real constant.

*In lecture we mentioned some difficulties with using this formula for $\mathbf{S}$ in a general situation, but it should be ok when talking about waves of a single frequency ω.

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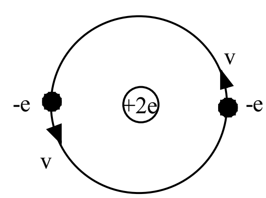
3) [35 points total]

Consider a classical model of the Helium atom as shown below. Two electrons, each with charge $-e$, orbit together on opposite sides of a nucleus of charge $+2e$. The radius of the orbit is a_0 and the electrons orbit with speed v, giving an angular velocity $\omega_0 = v/a_0$.



a) [6 pts] Compute the electric dipole moment $\mathbf{p}(t)$ and show that there is no electric dipole radiation.

b) [7 pts] Compute the magnetic dipole moment $\mathbf{m}(t)$ and show that there is no magnetic dipole radiation.

c) [10 pts] Compute the electric quadrupole tensor $\mathbf{Q}(t)$. What is its frequency of oscillation ω? Write your result in the form $\mathbf{Q}(t) = \mathbf{Q}_0 + \mathrm{Re}[\mathbf{Q}_\omega \mathrm{e}^{-i\omega t}]$, where $\mathbf{Q}_0$ and $\mathbf{Q}_\omega$ are constant tensors.

d) [6 pts] The amplitude of the vector potential oscillation for electric quadrupole radiation, in the radiation zone approximation, is given by

$$\mathbf{A}_\omega(\mathbf{r}) = -\frac{k^2 \mathrm{e}^{ikr}}{6r}\, \hat{\mathbf{r}} \cdot \mathbf{Q}_\omega$$

What are the amplitudes of the electric field $\mathbf{E}_\omega(\mathbf{r})$ and magnetic field $\mathbf{B}_\omega(\mathbf{r})$ oscillations in this radiation zone approximation? [Hint: recall, you can get $\mathbf{E}_\omega(\mathbf{r})$ from $\mathbf{B}_\omega(\mathbf{r})$ by using Ampere's law]

e) [6 pts] Using your results from parts (c) and (d), what is the angular distribution of the radiated power $dP/d\Omega$ in this electric quadrupole approximation? [Note: this is the hardest part algebraically; be sure to complete other parts of the exam first!]