PHY 415 **Midterm Exam** **Fall 2023**

This exam is closed book, closed notes. You may not consult with any other person or resource, except for the formulae sheet provided with this exam and your one page "cheat sheet." Please write clearly and use a dark pen or pencil. The better you explain your steps, the more likely you are to get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at.

Please write the academic honesty pledge, and sign your name, at the top of your work: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

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1) [35 points total]
Give a clear and convincing short answer to each of the following questions. An involved analytic calculation should not be needed.

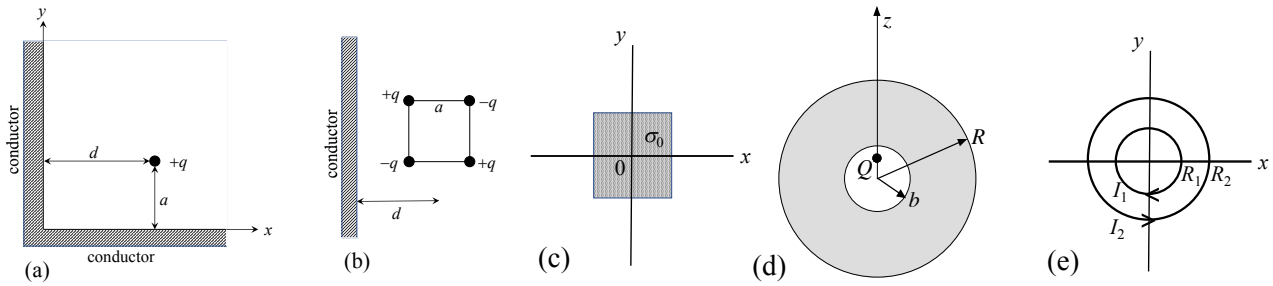
a) (7 pts) A charge $+q$ is placed a distance a above an infinite flat conductor in the xz plane, and a distance d in front of an infinite flat conductor in the yz plane, as shown in figure (a) below (the z axis points out of the page). How does $|\mathbf{E}(\mathbf{r})|$ decay as $\mathbf{r}$ gets very far from the charge?

b) (7 pts) Four charges, of equal magnitudes q but opposite signs, are placed at the corners of a square of side a, whose center is a distance d in front on an infinite flat conducting plane, as shown in figure (b) below. How does $|\mathbf{E}(\mathbf{r})|$ decay as $\mathbf{r}$ gets very far from the charges?

c) (7 pts) Consider a thin square of side length a, lying in the xy plane at $z = 0$, and centered about the origin, as in figure (c) below. The square carries a uniform surface charge σ_0. Consider the electric field $\mathbf{E}$ at a point at height z on the $z-$axis. Write an approximate expression for $\mathbf{E}(z)$ when $z \gg a$ and when $z \ll a$.

d) (7 pts) A conducting shpere of radius R is connected to a battery which keeps it at a constant potential ϕ_0 relative to a reference point at infinity (i.e. $\phi \to 0$ as $r \to \infty$). Inside the sphere is a concentric hollow spherical cavity of radius b, and inside the cavity is a charge Q, positioned a distance s from the origin, as shown in figure (d) below. Is there a net force on Q? If so, what is the physical origin of the force, and in what direction would it cause Q to move?

e) (7 pts) Consider two circular wire loops in the xy plane at $z = 0$, centered about the origin, as shown in figure (e) below. One has radius R_1 and carries a current I_1 flowing in a clockwise direction. The other has radius R_2 and carries a current I_2 flowing in the counterclockwise direction. What will be the magnetic field to leading order for positions far from the loops, $r \gg R_{1,2}$? Is there a particular case where your answer might be different?



2) [35 points total]

A thin spherical shell of radius R carries a surface charge density, $\sigma(\theta) = \sigma_0 \sin^2\theta$, where θ is the usual polar angle in spherical coordinates.

a) (25 pts) Find the electrostatic potential both inside the shell ($r < R$) and outside the shell ($r > R$).

b) (10 pts) Taking the origin at the center of the shell, does this configuration have a non-zero monopole moment? non-zero dipole moment? non-zero quadrupole moment? non-zero higher moments? You do not need to explicitly calculate these moments, but you must give a clear and convincing explanation for your answer (i.e. you cannot just say "yes" or "no").

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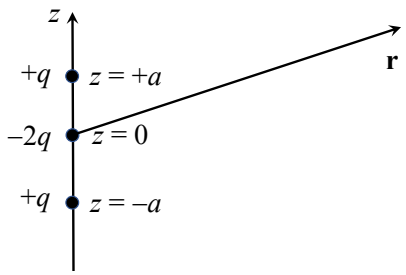
3) [30 points total]

Consider three charges arranged in a line along the $\hat{z}$ axis, as in the diagram below:



a) (10 pts) Compute the monopole, dipole, and quadrupole moments of this charge configuration, with respect to the origin as shown.

b) (20 pts) Find the electrostatic potential $\phi(\mathbf{r})$ at the observation point $\mathbf{r}$, using the multipole expansion through the quadrupole term. Express your answer in spherical coordinates, $\phi(r, \theta, \varphi)$.