Discussion Question 1.1

We know that, in general, Coulomb's law gives the solution to all electrostatic problems,

$$\mathbf{E}(\mathbf{r}) = k_1 \int d^3r' \rho(\mathbf{r}') \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \quad (1)$$

We are asked for a situation in which we would not apriori know the charge density ρ, and so could not apply the above to solve for $\mathbf{E}$.

One possible situation would be where we don't know ρ because the distribution of charges is too complex to be easily specified. For example, an electric field inside some material would be due in part to the charges from all the electrons and protons of the atoms and molecules making up that material. The charge density would be some ρ that varies very rapidly on the atomic length scale, and one might think that it would be too difficult to specify such a ρ. We will see how to deal with such situations in units 3 and 5. Moreover, if one had a material, such as a crystal, in which the location of the atoms is known, one could in principle use quantum mechanics to solve for the electron distribution in the system and so define a ρ and try to compute (with fancy supercomputers if need be) the resulting $\mathbf{E}$.

A more interesting situation may arise because we may not know exactly where the charges are. Charges will feel a force from the electric field, which may cause them to move from where they were to new positions determined by the electric field, which itself is determined in part by where those charges are! The electric field and the charge density must be solved for together in some self-consistent way. An example you should all be familiar with is that of a conductor. When there is no electric field, the distribution of conduction electrons is spatially uniform (when averaging on a length scale large compared to the interatomic distances), so as to cancel out the charge from the protons of the ions in their lattice positions. The negative electric charge from the electrons cancels the positive electric charge from the protons, and so there is no net electric field. But if an external electric field is applied to the conductor, we know (and will review in Notes 2-1) that the electric field inside the conductor must stay equal to zero. If the electric field inside the conductor was not zero, it would exert forces on the conduction electrons that would cause them to move, and give electric currents, and we would not be in a static situation. So what happens? The force on the conduction electrons from the external electric field causes the conduction electrons to move from their previously uniform spatial distribution, to new positions, so that the total field from the applied electric field *plus* the field produced by the conduction electrons sums to zero inside the conductor. What is the ρ inside the conductor that causes this to happen? This has to be solved self-consistently along with solving for the total electric field. We will see in unit 2 that the way to do this is in terms of a "boundary value problem," applied to the electrostatic differential equations for $\mathbf{E}$.