PHY 415 **Endterm Exam** **Fall 2024**

This exam is closed book, closed notes. You may not consult with any other person or resource, except for the formulae sheet provided with this exam and your one page "cheat sheet." Please write clearly and use a dark pen or pencil. The better you explain your steps, the more likely you are to get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at.

Please write the academic honesty pledge, and sign your name, at the top of your work: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

---

1) [30 points total - 6 points each part]

Give a brief and to the point answer to each of the following. You may cite appropriate equations when it helps to explain a point, but no calculations should be necessary.

a) In a conductor or a non-conducting dielectric, the dielectric function $\varepsilon(\omega)$ is in general a complex valued, frequency dependent, function. Give two consequences of the fact that $\varepsilon(\omega)$ varies with ω. Give two consequences of the fact that $\varepsilon(\omega) = \varepsilon_1(\omega) + i\varepsilon_2(\omega)$ may have an imaginary component.

b) Describe one qualitative difference between the propagation of a plane polarized simple-harmonic (i.e. single frequency) electromagnetic wave in a conductor vs in a non-conducting dielectric.

c) Describe one physical phenomenon associated with the plasma frequency of a conductor.

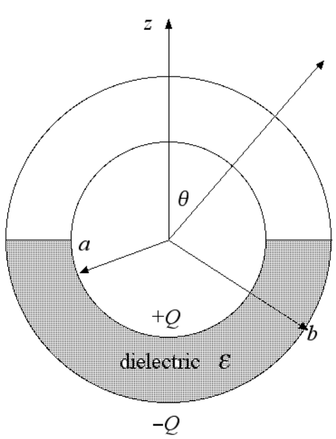
d) What is Snell's law? In what case is Snell's law not valid?

e) What happens at Brewster's angle?

---

2) [35 points total]

Two thin concentric conducting spherical shells of inner radius a and outer radius b carry total free charges $+Q$ and $-Q$ respectively. The space between them is half filled by a dielectric material with dielectric constant ε, as sketched below.



a) [15 pts] Find the electric field **E** everywhere between the shells, $a < r < b$.

b) [10 pts] Calculate the total surface charge density $\sigma_{\text{tot}}(\theta)$ on the surface of the inner shell at $r = a$.

c) [10 pts] Calculate the induced bound surface charge density $\sigma_b(\theta)$ on the surface of the dielectric at $r = a$.

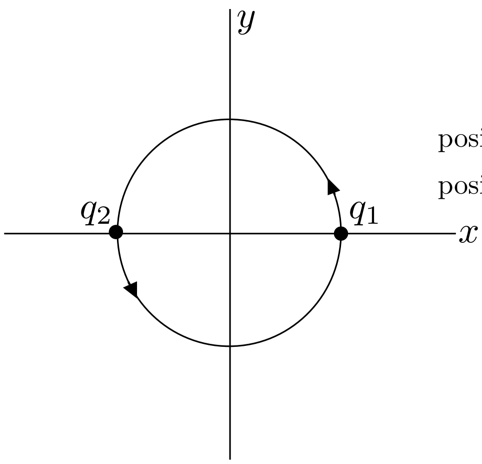
3) [35 points total]

For a time dependent charge distribution with an oscillating electric dipole moment $\mathbf{p}(t) = \mathrm{Re}[\mathbf{p}_\omega \mathrm{e}^{-i\omega t}]$, the radiated fields (i.e. in the radiation zone) in the electric dipole approximation are given by,

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left[-k^2\, \frac{\mathrm{e}^{i(kr-\omega t)}}{r}\, \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{p}_\omega)\right], \quad \mathbf{B}(\mathbf{r},t) = \mathrm{Re}\left[k^2\, \frac{\mathrm{e}^{i(kr-\omega t)}}{r}\, \hat{\mathbf{r}} \times \mathbf{p}_\omega\right],$$

where $k = \omega/c$.

Consider two charges q_1 and q_2 which are moving in a circular orbit of radius d in the xy plane. The charges are located on opposite sides of the orbit, and the rotate with angular frequency ω, as in the sketch below.



position of q_1 is: $\mathbf{r}_1 = d\cos(\omega t)\hat{\mathbf{x}} + d\sin(\omega t)\hat{\mathbf{y}}$

position of q_2 is: $\mathbf{r}_2 = -d\cos(\omega t)\hat{\mathbf{x}} - d\sin(\omega t)\hat{\mathbf{y}}$

Assume $q_1 = -q_2$.

a) [15 pts] Find the radiated $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ fields in the electric dipole approximation. Express your answers as *real valued* functions of space and time.

b) [15 pts] Find the time averaged Poynting vector $\langle \mathbf{S}(\mathbf{r}) \rangle$ as a function of space. Express your final answer in terms of polar coordinates.

c) [5 pts] Sketch a polar plot of the angular distribution of the radiated power $dP/d\Omega$.