PHYS 415 Solutions Problem Set 4

1) **Discussion Question 2.4**

The question is, if the monopole moment $q = 0$, and the dipole moment $\mathbf{p} \neq 0$, can we make the quadrupole moment $\overleftrightarrow{\mathbf{Q}}$ vanish by taking an appropriate origin for the coordinate system?

When $q = 0$, the quadrupole tensor in the translated coordinate system is,

$$\tilde{\overleftrightarrow{\mathbf{Q}}} = \overleftrightarrow{\mathbf{Q}} - 3\mathbf{pd} - 3\mathbf{dp} + 2(\mathbf{p}\cdot\mathbf{d})\overleftrightarrow{\mathbf{I}} \tag{1}$$

We assume that in the initial coordinates $\overleftrightarrow{\mathbf{Q}}$ is diagonal, so $Q_{xy} = Q_{yx} = Q_{yz} = Q_{zy} = Q_{zx} = Q_{xz} = 0$, but Q_{xx} and $Q_{yy} \neq 0$. Because $\overleftrightarrow{\mathbf{Q}}$ must be traceless, $Q_{zz} = -(Q_{xx} + Q_{yy})$.

Now we want to make a translation by $\mathbf{d}$ and see if we can get $\tilde{\overleftrightarrow{\mathbf{Q}}} = 0$. Since $\tilde{\overleftrightarrow{\mathbf{Q}}}$ is symmetric and traceless, we would need,

$$\tilde{Q}_{xx} = Q_{xx} - 6p_x d_x + 2\mathbf{p}\cdot\mathbf{d} = 0 \tag{2}$$

$$\tilde{Q}_{yy} = Q_{yy} - 6p_y d_y + 2\mathbf{p}\cdot\mathbf{d} = 0 \tag{3}$$

$$\tilde{Q}_{xy} = -3p_x d_y - 3p_y d_x = 0 \tag{4}$$

$$\tilde{Q}_{yz} = -3p_y d_z - 3p_z d_y = 0 \tag{5}$$

$$\tilde{Q}_{zx} = -3p_z d_x - 3p_x d_z = 0 \tag{6}$$

The above is 5 equations with only 3 unknowns, d_x, d_y, and d_z. In general the system is *overspecified* and there is no solution.

Note, even though $\tilde{\overleftrightarrow{\mathbf{Q}}}$ was diagonal in the original coordinate system, in general it will not remain diagonal in the translated coordinate system.

<u>**A special case**</u> where we can make $\tilde{\overleftrightarrow{\mathbf{Q}}} = 0$ is when the system has *rotational symmetry about an axis*, say the $\hat{\mathbf{z}}$ axis. Then by symmetry, the dipole moment $\mathbf{p}$ must be aligned parallel to the $\hat{\mathbf{z}}$ axis. If that is not obvious, one can see that mathematically as follows:

$$p_x = \int d^3r \rho(x,y,z)x = \int d^3r \rho(-x,-y,z)(-x) = \int d^3r \rho(x,y,z)(-x) = -p_x \qquad \Rightarrow \qquad p_x = 0 \tag{7}$$

In the first step above we just changed integration variables $x \leftrightarrow -x$ and $y \leftrightarrow -y$; in the second step we used that ρ is rotation symmetric about the $\hat{\mathbf{z}}$ axis, so by 90° rotation, $\rho(-x,-y,z) = \rho(x,y,z)$. Since $p_x = -p_x$, we must have $p_x = 0$, and similarly $p_y = 0$, so $\mathbf{p} = p\,\hat{\mathbf{z}}$.

Similarly we can show that the quadrupole moment $\overleftrightarrow{\mathbf{Q}}$ can be diagonalized with $\hat{\mathbf{z}}$ as one of the eigen-directions. Consider the off-diagonal term Q_{xz},

$$Q_{xz} = \int d^3r\, \rho(x,y,z)\,(3xz) = \int d^3r\, \rho(-x,-y,z)\,(-3xz) = \int d^3r\, \rho(x,y,z)(-3xz) = -Q_{xz} \quad \Rightarrow \quad Q_{xz} = 0 \tag{8}$$

where the steps are the same as we used for the dipole moment p_x. Similarly, $Q_{zx} = Q_{yz} = Q_{zy} = 0$, so $\overleftrightarrow{\mathbf{Q}}$ has the form,

$$\overleftrightarrow{\mathbf{Q}} = \begin{pmatrix} Q_{xx} & Q_{xy} & 0 \\ Q_{ys} & Q_{yy} & 0 \\ 0 & 0 & Q_{zz} \end{pmatrix} \tag{9}$$

Since the $x - y$ subspace of this tensor is symmetric, it can be diagonalized by an appropriate rotation about the $\hat{\mathbf{z}}$ axis. But because the charge distribution has rotational symmetry about the $\hat{\mathbf{z}}$ axis, it must be that such a rotation leaves $\overleftrightarrow{\mathbf{Q}}$ invariant. We thus conclude that $Q_{xy} = Q_{yx} = 0$, and $Q_{xx} = Q_{yy} \equiv Q_\perp$.

We could see this directly as follows. If we call $\mathbf{r}_\perp = (x, y)$, the component of $\mathbf{r}$ in the xy plane, then by rotational symmetry, ρ can depend only on $r_\perp = |\mathbf{r}_\perp|$ and z. So, converting to cylindrical coordinates, we have,

$$Q_{xy} = \int d^3r\, \rho(r_\perp, z)\,(3xy) = \int_{-\infty}^{\infty} dz \int_0^{\infty} dr_\perp\, r_\perp \int_0^{2\pi} d\varphi\, \rho(r_\perp, z)(3r_\perp^2 \cos\varphi \sin\varphi) \tag{10}$$

Since $\int_0^{2\pi} d\varphi \cos\varphi \sin\varphi = 0$, we have $Q_{xy} = 0$ and similarly $Q_{yx} = 0$. And,

$$Q_{xx} = \int d^3r\, \rho(r_\perp, z)\,(3x^2 - r^2) = \int_{-\infty}^{\infty} dz \int_0^{\infty} dr_\perp\, r_\perp \int_0^{2\pi} d\varphi\, \rho(r_\perp, z)(3r_\perp^2 \cos^2\varphi - [r_\perp^2 + z^2]) \tag{11}$$

$$Q_{yy} = \int d^3r\, \rho(r_\perp, z)\,(3y^2 - r^2) = \int_{-\infty}^{\infty} dz \int_0^{\infty} dr_\perp\, r_\perp \int_0^{2\pi} d\varphi\, \rho(r_\perp, z)(3r_\perp^2 \sin^2\varphi - [r_\perp^2 + z^2]) \tag{12}$$

Since $\int_0^{2\pi} d\varphi\, \cos^2\varphi = \int_0^{2\pi} \sin^2\varphi = \pi$, we have that $Q_{xx} = Q_{yy}$.

So now we see that, with $\hat{\mathbf{z}}$ as an axis of rotational symmetry, $\overleftrightarrow{\mathbf{Q}}$ must have the form,

$$\overleftrightarrow{\mathbf{Q}} = \begin{pmatrix} Q_\perp & 0 & 0 \\ 0 & Q_\perp & 0 \\ 0 & 0 & Q_{zz} \end{pmatrix} \tag{13}$$

Using these results, that $p_x = p_y = 0$, and $Q_{xx} = Q_{yy} \equiv Q_\perp$, Eqs. (2)–(6) become,

$$\tilde{Q}_{xx} = Q_\perp + 2p\hat{\mathbf{z}} \cdot \mathbf{d} = 0 \tag{14}$$

$$\tilde{Q}_{yy} = Q_\perp + 2p\hat{\mathbf{z}} \cdot \mathbf{d} = 0 \tag{15}$$

$$\tilde{Q}_{xy} = 0 \tag{16}$$

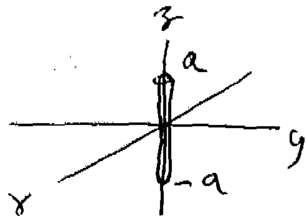
$$\tilde{Q}_{yz} = -3pd_y = 0 \tag{17}$$

$$\tilde{Q}_{zx} = -3pd_x = 0 \tag{18}$$

So if we now take $\mathbf{d}$ to be parallel to $\mathbf{p}$, then $d_x = d_y = 0$, and the off-diagonal terms of $\tilde{\overleftrightarrow{\mathbf{Q}}}$ vanish. We can then make the diagonal terms vanish by taking,

$$\mathbf{d} = \frac{-Q_\perp}{2p}\,\hat{\mathbf{z}} \tag{19}$$

and we get $\tilde{\overleftrightarrow{\mathbf{Q}}} = 0$.

2)  $\lambda(z)$ line charge density

(5)

Note we can represent the charge by the following volume charge density:

$$\rho(\vec{r}) = \lambda(z)\,\delta(x)\,\delta(y)$$

where the δ-functions act to confine the charge to the z-axis.

<u>monopole moment</u>: $q = \int d^3r\, \rho(\vec{r}) = \int dx\,dy\,dz\, \lambda(z)\,\delta(x)\,\delta(y)$

$$\boxed{q = \int dz\, \lambda(z)}$$

<u>dipole moment</u> $p_i = \int d^3r\, \rho(\vec{r})\, r_i = \int dx\,dy\,dz\, \lambda(z)\,\delta(x)\,\delta(y)\, r_i$

$\Rightarrow$ $p_x = \int dy\,dz\, \lambda(z)\,\delta(y) \int dx\, \delta(x)\, x = 0$

$p_y = \int dx\,dz\, \lambda(z)\,\delta(x) \int dy\, \delta(y)\, y = 0$

$p_z = \int dz\, \lambda(z)\, z \int dx\, \delta(x) \int dy\, \delta(y)$

$$\boxed{p_z = \int dz\, \lambda(z)\, z \quad , \; p_x = p_y = 0}$$

using $\int dx\, \delta(x - x_0)\, f(x) = f(x_0)$

here $x_0 = 0$, and $f(x) = x$

so $\int dx\, \delta(x)\, x = 0$.

<u>quadrapole tensor</u>

$$Q_{ij} = \int d^3r\, \rho(\vec{r}) \left[3r_i r_j - r^2 \delta_{ij}\right] = \int dx\,dy\,dz\, \lambda(z)\delta(x)\delta(y) \times \left[3r_i r_j - r^2\delta_{ij}\right]$$

$$Q_{xx} = \int dx \int dy \int dz\, \lambda(z)\,\delta(x)\,\delta(y)\left[3x^2 - (x^2+y^2+z^2)\right]$$

$$= \int dz\, \lambda(z)\,(-z^2) \qquad \text{after doing } \int dx\, \delta(x)\, x^2 = 0,\ \int dy\, \delta(y)\, y^2 = 0$$

$$Q_{xy} = \int dx\,dy\,dz\, \lambda(z)\,\delta(x)\,\delta(y)\,[3xy] = 0$$

since $\int dx\, \delta(x)\, x = 0$

Similarly $Q_{xz} = Q_{yz} = 0$

$$Q_{yy} = \int dx\,dy\,dz \left[3y^2 - (x^2+y^2+z^2)\right]\lambda(z)\delta(x)\delta(y)$$

$$= \int dz\, \lambda(z)\,(-z^2)$$

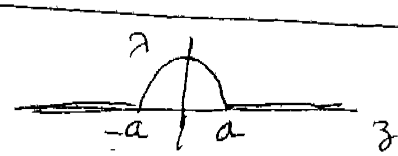
$$Q_{zz} = \int dx\,dy\,dz\, \lambda(z)\delta(x)\delta(y)\left[3z^2 - (x^2+y^2+z^2)\right]$$

$$= \int dz\, \lambda(z)\, 2z^2$$

$$\overleftrightarrow{Q} = \begin{pmatrix} -\int dz\, \lambda(z)\, z^2 & 0 & 0 \\ 0 & -\int dz\, \lambda(z)\, z^2 & 0 \\ 0 & 0 & 2\int dz\, \lambda(z)\, z^2 \end{pmatrix}$$

$$\boxed{\overleftrightarrow{Q} = \int dz\, \lambda(z) z^2 \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}}$$

a) $\lambda = \lambda_0 \cos\left(\frac{\pi z}{2a}\right)$



$$q = \int_{-a}^{a} dz\, \lambda(z) = \int_{-a}^{a} dz\, \lambda_0 \cos\left(\frac{\pi z}{2a}\right) = \left[\frac{\lambda_0 \sin\left(\frac{\pi z}{2a}\right)}{(\pi/2a)}\right]_{-a}^{a}$$

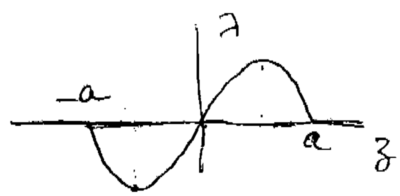
$$= \frac{2a\lambda_0}{\pi}\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right] = \frac{4a\lambda_0}{\pi}$$

$q > 0$ (as is obvious from sketch of $\lambda(z)$ vs. z)

So leading term is the monopole term

$$\boxed{\phi \simeq \frac{q}{r} = \frac{\frac{4a\lambda_0}{\pi}}{r}}$$

b) $\lambda = \lambda_0 \sin(\pi z/a)$



since $\lambda(z)$ antisymmetric about $z = 0$, it is clear that $\int_{-a}^{a} dz\, \lambda(z) = 0$, so

monopole moment $q = 0$.

dipole moment $\quad P_z = \int_{-a}^{a} dz\, \lambda(z)\, z$

now integrand is symmetric about the origin and so will not vanish. $\Rightarrow$ dipole term is leading term in multipole expansion.

$$P_z = \lambda_0 \int_{-a}^{a} dz\; z \sin\left(\frac{\pi z}{a}\right) = \left[\frac{-\lambda_0\, z \cos\left(\frac{\pi z}{a}\right)}{\left(\frac{\pi}{a}\right)} \right]_{-a}^{a} + \lambda_0 \int_{-a}^{a} dz \frac{\cos\left(\frac{\pi z}{a}\right)}{\frac{\pi}{a}}$$

integrate by parts

$$= -\frac{a\lambda_0}{\pi}\Big[\, \underbrace{a\cos(\pi)}_{=-1} + \underbrace{a\cos(-\pi)}_{=-1} \Big] + \frac{a\lambda_0}{\pi}\left[\frac{\sin\left(\frac{\pi z}{a}\right)}{\left(\frac{\pi}{a}\right)} \right]_{-a}^{a}$$

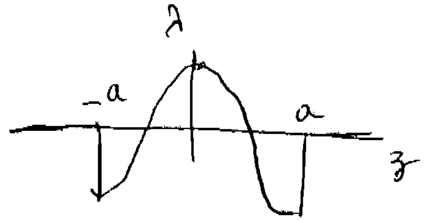
$$= \frac{2a^2\lambda_0}{\pi} + \frac{a^2\lambda_0}{\pi^2}\Big[\underbrace{\sin\pi}_{=0} - \underbrace{\sin(-\pi)}_{=0} \Big]$$

$$= \frac{2a^2\lambda_0}{\pi}$$

$$\phi \simeq \frac{\vec{p}\cdot\hat{r}}{r^2} = \frac{\frac{2a^2\lambda_0}{\pi}}{r^2}\,\hat{z}\cdot\hat{r}$$

$$\boxed{\phi = \frac{\frac{2a^2\lambda_0}{\pi}}{r^2}\cos\theta}$$

c) $\lambda = \lambda_0 \cos(\pi z/a)$



from sketch we can see that $\int dz\, \lambda(z) = 0$

also $\int dz\, \lambda(z) z = 0$ as integrand is antisymmetric about $z = 0$

so $\underline{q = \vec{p} = 0}$

For $\int dz\, \lambda(z) z^2$, integrand is symmetric about $z=0$ and so doesn't vanish

$\Rightarrow$ leading term is <u>quadrapole term</u>.

$$\int_{-a}^{a} dz\, \lambda(z) z^2 = \lambda_0 \int_{-a}^{a} dz\, z^2 \cos\left(\frac{\pi z}{a}\right)$$ ← look up in integral tables, or do two integration by parts

$$= \lambda_0 \left[\frac{2z}{(\pi/a)^2} \cos\frac{\pi z}{a} + \left(\frac{z^2}{(\pi/a)} - \frac{2}{(\pi/a)^3} \right) \sin\left(\frac{\pi z}{a}\right) \right]_{-a}^{a}$$

$$= \lambda_0 \left[\frac{2a^3}{\pi^2} \underbrace{\cos\pi}_{=-1} + \frac{2a^3}{\pi^2} \underbrace{\cos(-\pi)}_{=-1} \right.$$

$$\left. + \left(\frac{a^3}{\pi} - \frac{2a^3}{\pi^3} \right) \underbrace{\sin(\pi)}_{=0} - \left(\frac{a^3}{\pi} - \frac{2a^3}{\pi^3} \right) \underbrace{\sin(-\pi)}_{=0} \right]$$

$$= -\frac{4a^3}{\pi^2}\lambda_0$$

$$\hat{r}\cdot\begin{pmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 2\end{pmatrix}\cdot\hat{r} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)\cdot\begin{pmatrix}-\sin\theta\cos\phi\\ -\sin\theta\sin\phi\\ 2\cos\theta\end{pmatrix}$$

$$= -\sin^2\theta\cos^2\phi - \sin^2\phi\sin^2\phi + 2\cos^2\theta$$

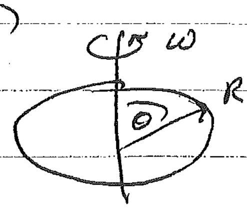
$$= 2\cos^2\theta - \sin^2\theta = 3\cos^2\theta - 1$$

$$\boxed{\phi = \frac{\frac{1}{2}\,\hat{r}\cdot\overleftrightarrow{Q}\cdot\hat{r}}{r^3} = \frac{-\frac{4\lambda a^3}{\pi^2}\,(3\cos^2\theta - 1)}{2r^3}}$$

$=$

3)

a)



$\vec{\omega} = \omega \hat{z}$

surface current

$\vec{K}(\theta) = \sigma \underbrace{\vec{\omega} \times \vec{r}}_{\vec{v}}$ $\vec{r}$ on surface of sphere

$= \sigma \omega R \sin\theta \, \hat{\varphi}$

magnetic dipole moment

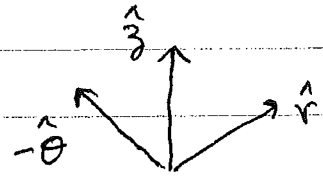
$$\vec{m} = \frac{1}{2c} \int d^3r \; \vec{r} \times \vec{K}(\vec{r})$$

$$= \frac{1}{2c} \oint_S da \; \vec{r} \times \vec{K}(\vec{r})$$

$$= \frac{1}{2c} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \, R^2 \, \underbrace{(R\hat{r})}_{\vec{r}} \times \underbrace{(\sigma \omega R \sin\theta \, \hat{\varphi})}_{\vec{K}}$$

$$= -\frac{\omega \sigma R^4}{2c} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \; \sin^2\theta \; \hat{\theta}$$

Since $\hat{\theta}$ varies with position, the integral is easier to do if we rewrite in Cartesian basis.



$$-\hat{\theta} = \sin\theta \, \hat{z} - \cos\theta \cos\varphi \, \hat{x} - \cos\theta \sin\varphi \, \hat{y}$$

when integrate over φ, the $\hat{x}$ and $\hat{y}$ pieces vanish.

$$\vec{m} = \frac{\omega \sigma R^4}{2c} \hat{z} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \; \sin^3\theta$$

$$\vec{m} = \frac{\pi}{c} \omega \sigma R^4 \hat{z} \int_0^{\pi} d\theta \, \sin\theta \, (1-\cos^2\theta)$$

$$= \frac{\pi \omega \sigma R^4}{c} \hat{z} \left[-\cos\theta + \frac{\cos^3\theta}{3} \right]_0^{\pi}$$

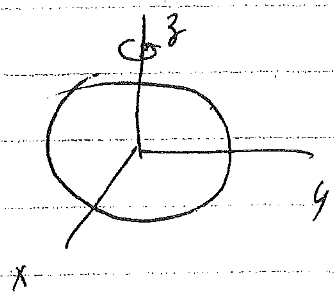
$$\boxed{\vec{m} = \frac{4\pi \omega \sigma R^4}{3c} \hat{z}}$$

$$\vec{A} = \frac{\vec{m} \times \hat{r}}{r^2} = \frac{m \sin\theta \, \hat{\varphi}}{r^2}$$ in spherical coords with $\vec{m} \parallel \hat{z}$

$$\vec{B} = \frac{m \left[2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta} \right]}{r^3}$$ in spherical coords with $\vec{m} \parallel \hat{z}$

$$\boxed{\vec{B} = \frac{4\pi \omega \sigma R^4}{3c} \, \frac{(2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta})}{r^3}}$$

b)



rotating sphere $\vec{K} = K(\theta)\,\hat{\varphi}$

where $K(\theta) = \sigma \omega R \sin\theta$

$$\vec{B} = -\vec{\nabla}\phi_M$$

$$\vec{\nabla}\cdot\vec{B} = -\nabla^2 \phi_M = 0$$

$$\phi_M(\vec{r}) = \begin{cases} \sum_\ell A_\ell r^\ell P_\ell(\cos\theta) & r<R \\ \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta) & r>R \end{cases}$$

$$\vec{B} = -\frac{\partial \phi_M}{\partial r}\hat{r} - \frac{1}{r}\frac{\partial \phi_M}{\partial \theta}\hat{\theta}$$

$$= \begin{cases} \sum_\ell \left\{ -\ell A_\ell r^{\ell-1} P_\ell(\cos\theta)\,\hat{r} + A_\ell r^{\ell-1}\sin\theta\, P_\ell'(\cos\theta)\,\hat{\theta} \right\} & r \\ \sum_\ell \left\{ \frac{(\ell+1)B_\ell}{r^{\ell+2}} P_\ell(\cos\theta)\,\hat{r} + \frac{B_\ell}{r^{\ell+2}}\sin\theta\, P_\ell'(\cos\theta)\,\hat{\theta} \right\} & r>R \end{cases}$$

where we used $\frac{\partial P_\ell(\cos\theta)}{\partial\theta} = \frac{\partial P_\ell(x)}{\partial x}\frac{\partial x}{\partial\theta} = -\sin\theta\, P_\ell'(\cos\theta)$

where $P_\ell' = \frac{dP_\ell(x)}{dx}$

boundary condition

$$\vec{B}_{above} - \vec{B}_{below} = \frac{4\pi}{c}\vec{K}\times\hat{n} = \frac{4\pi}{c}K(\theta)(\hat{\varphi}\times\hat{r})$$

$$= \frac{4\pi}{c}K(\theta)\,\hat{\theta}$$

$$\Rightarrow \left(\vec{B}_{above} - \vec{B}_{below}\right)\cdot\hat{r} = 0$$

$$\Rightarrow \left[\frac{(\ell+1)B_\ell}{R^{\ell+2}} + \ell A_\ell R^{\ell-1}\right] = 0 \Rightarrow B_\ell = -\left(\frac{\ell}{\ell+1}\right) A_\ell R^{2\ell+1}$$

$$\Rightarrow \left(\vec{B}_{above} - \vec{B}_{below}\right)\cdot\hat{\theta} = \frac{4\pi K(\theta)}{c}$$

$$\Rightarrow \sum_\ell \left(\frac{B_\ell}{R^{\ell+2}} - A_\ell R^{\ell-1}\right)\sin\theta\, P_\ell'(\cos\theta)$$

$$= -\sum_\ell \left(\left[\left(\frac{\ell}{\ell+1}\right)+1\right] A_\ell R^{\ell-1}\right)\sin\theta\, P_\ell'(\cos\theta)$$

$$= -\sum_\ell \left(\frac{2\ell+1}{\ell+1}\right) R^{\ell-1} A_\ell \sin\theta\, P_\ell'(\cos\theta) = \frac{4\pi}{c}\sigma\omega R\sin\theta$$

$$\Rightarrow A_\ell = 0 \quad \text{except for } \ell=1 \text{ as } P_1'(x) = 1$$

$$A_1 = -\frac{4\pi\sigma\omega R}{c}\left(\frac{2}{3}\right)\frac{1}{R^0} = -\left(\frac{2}{3}\right)\frac{4\pi\sigma\omega R}{c}$$

$$\Rightarrow B_1 = \frac{4\pi\sigma\omega R^4}{3c}$$

$\Rightarrow$ <u>inside $r < R$</u>

$$\vec{B}(\vec{r}) = \left(\frac{2}{3}\right)\frac{4\pi\sigma\omega R}{c}\cos\theta\,\hat{r} - \left(\frac{2}{3}\right)\frac{4\pi\sigma\omega R}{c}\sin\theta\,\hat{\theta}$$

$$= \frac{8\pi\sigma\omega R}{3c}\left(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\right)$$

$$\boxed{\vec{B}(\vec{r}) = \frac{8\pi\sigma\omega R}{3c}\hat{z}} \qquad \hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

outside $r > R$

$$\vec{B}(\vec{r}) = (2)\,\frac{4}{3c}\pi\sigma\omega\frac{R^4}{r^3}\cos\theta\,\hat{r} + \frac{4}{3}\pi\sigma\omega\frac{R^4}{r^3}\sin\theta\,\hat{\theta}$$

$$\boxed{\vec{B}(\vec{r}) = \left(\frac{4}{3c}\pi\sigma\omega R^4\right)\frac{2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}}{r^3}}$$ same as in part (a)

field of a point dipole with $\vec{m} = \frac{4}{3}\frac{\pi}{c}\sigma\omega R^4\,\hat{z}$

easy to check that $\vec{B}\cdot\hat{r}$ is continuous