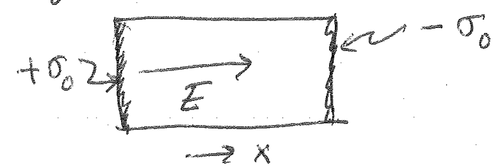
PHY 415 Solutions Problem Set 8

1) a) surface charge is $\sigma = \begin{cases} -eN\delta x & \text{on right side} \\ +eN\delta x & \text{on left side} \end{cases}$

where N is the density of conduction electrons

b) This is just like parallel place capacitor with $+\sigma_0$ on one plate and $-\sigma_0$ on other plate. The field inbetween is therefore

$$\vec{E} = 4\pi\sigma_0 \hat{x} = 4\pi e N \delta x \ \hat{x}$$



where $\hat{x}$ is direction pointing from left to right

c) total force on electrons $=$ (total charge) $\vec{E}$

$= -eNV\vec{E}$ where V is volume

total mass of electrons is $M = mNV$

Newton's equation of motion

$$\vec{F} = M\vec{a} \Rightarrow -eNV\vec{E} = mNV\vec{a}$$

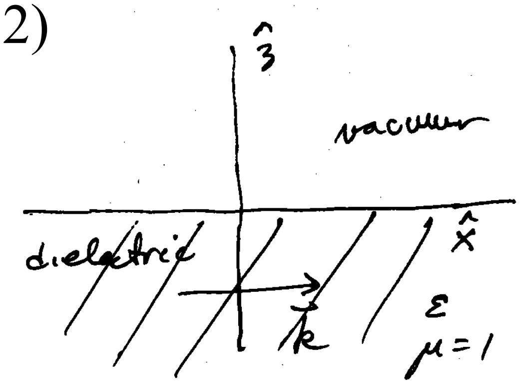
$$-\frac{e\vec{E}}{m} = \vec{a} \qquad \vec{a} = \delta\ddot{x}\,\hat{x}$$

$$\Rightarrow \delta\ddot{x} = -4\pi\frac{e^2}{m}N\ \delta x$$

$\Rightarrow$ simple harmonic motion with frequency

$\omega_p = \sqrt{\dfrac{4\pi e^2 N}{m}}$ this is the plasma freq.

2)



in dielectric:
wavevector $\vec{k} = k\hat{x}$
frequency ω

in vacuum:
wavevector $\vec{k}'$
frequency ω'

a) boundary conditions on fields:
tangential components $\vec{E}$ and $\vec{H}$ continuous
normal components $\vec{D}$ and $\vec{B}$ continuous

These conditions $\Rightarrow \begin{cases} \omega = \omega' & \rightarrow \text{freqs equal} \\ k_x = k_x' \\ k_y = k_y' \end{cases}$ $\rightarrow$ components of wavevector in the plane of interface are equal

b) wave vectors are related to frequency by dispersion relation

in dielectric $|\vec{k}|^2 = \frac{\omega^2}{c^2}\varepsilon$

in vacuum $|\vec{k}'|^2 = \frac{\omega^2}{c^2}$

dielectric $\vec{k} = k\hat{x} \rightarrow k^2 = \frac{\omega^2}{c^2}\varepsilon$

vacuum $\vec{k}' = k_x'\hat{x} + k_y'\hat{y} + k_z'\hat{z}$

boundary conditions give $k_x' = k$, $k_y' = 0$

$\vec{k}' = k\hat{x} + k_z'\hat{z}$

$$\Rightarrow \quad |\vec{k}'|^2 = k^2 + (k_z')^2 = \frac{\omega^2}{c^2}$$

$$\Rightarrow \quad (k_z')^2 = \frac{\omega^2}{c^2} - k^2 = \frac{\omega^2}{c^2} - \frac{\omega^2}{c^2}\varepsilon$$

$$(k_z')^2 = \frac{\omega^2}{c^2}(1-\varepsilon) < 0 \quad \text{since } \varepsilon > 1$$

$\Rightarrow k_z'$ is pure imaginary

$$\boxed{k_z' = \frac{\omega}{c}\sqrt{\varepsilon - 1}\, i}$$

so in the vacuum outside the dielectric the wave decays exponentially as one moves away from the surface

$$E_{vac} \sim e^{i(\vec{k}\cdot r - \omega t)} = e^{i(kx - \omega t)} e^{-|k_z'| z}$$