PHY 418 **Midterm Exam** **Spring 2007**

1) [50 points total]

Consider a system in the canonical ensemble. Let $U \equiv \langle E \rangle$ be the total average energy, and $\Delta E \equiv E - U$ be the fluctuation away from this average. [Warning: only part (c) refers to an ideal gas; parts (a) and (b) refer to any general system.]

a) [15 pts] Derive the following expression between the specific heat at constant volume, C_V, and fluctuations in the total energy E,

$$C_V = \frac{1}{k_B T^2}\left[\langle E^2 \rangle - \langle E \rangle^2\right] = \frac{1}{k_B T^2}\langle (\Delta E)^2 \rangle \tag{1}$$

b) [20 pts] Show that

$$\langle (\Delta E)^3 \rangle = k_B^2 T^4 \left(\frac{\partial C_V}{\partial T}\right)_V + 2k_B^2 T^3 C_V \tag{2}$$

c) [15 pts] For a classical, non-relativistic, non-interacting ideal gas of N indistinguishable particles, show that,

$$\left\langle \left(\frac{\Delta E}{U}\right)^2 \right\rangle = \frac{2}{3N}, \quad \text{and} \quad \left\langle \left(\frac{\Delta E}{U}\right)^3 \right\rangle = \frac{8}{9N^2} \tag{3}$$

2) [50 points total]

Consider a classical gas of N indistinguishable, non-interacting, particles with *ultra-relativistic* energies, i.e. the energy - momentum relation of a particle is given by $\epsilon(\mathbf{p}) = |\mathbf{p}|c$, with c the speed of light and $\mathbf{p}$ the particle's momentum. The gas is in equilibrium at temperature T, confined to a three dimensional box of volume V.

a) [20 pts] Compute the *canonical* partition function for this system.

b) [15 pts] Show that this system obeys the usual ideal gas law, $pV = Nk_BT$.

c) [15 pts] Find the specific heat at constant *pressure*, C_p.