PHYS 418 **Endterm Exam** **Spring 2024**

For this exam you may use one 8.5"x11" sheet of paper on which you have written notes on one side of the page. No other books, notes, or resources are permitted. Please write clearly with a dark pen or pencil. The better you explain the steps you make in your solutions, the more likely it is that you can get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at.

Please write the academic honesty pledge, and **sign** your name, at the top of your work:
I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

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1) [35 points total]

Consider of box of volume V containing two different ideal gases A and B, which consist of classical, non-relativisitic, indistinguishable particles. The N_A particles of gas A may be considered as point particles with mass m_A and no internal degrees of freedom. The N_B particles of gas B have mass m_B and contain an internal degree of freedom which can have two possible energy values, 0 or Δ, where $\Delta > 0$. The two gases are in equilibrium with thermal and particle reservoirs that fix the temperature at T, and chemical potentials of the two gases at μ_A and μ_B.

a) [18 pts] Working in the *grand canonical ensemble*, find the ratio of particles N_A/N_B and show how it depends on T, μ_A, and μ_B.

b) [17 pts] Suppose the system is now isolated from the particle reservoirs, and the particles can react via the chemical equation $2A \leftrightarrow B$. If initially there are N_0 particles A and no particles B. What will be the ratio N_A/N_B when the system comes to equilibrium?

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2) [30 points]

Consider an ideal gas of N spin 1/2 non-relativisitic fermions, confined to move in a one dimensional system in an external harmonic potential $U(x) = \frac{1}{2}m\omega_0^2 x^2$. You may assume that N is large.

a) [5 pts] If the number of single particle states between energy ε and $\varepsilon + d\varepsilon$ is $g(\varepsilon)d\varepsilon$, then find the density of states per unit energy, $g(\varepsilon)$. You may assume $\varepsilon \gg \hbar\omega_0$.

b) [5 pts] What is the Fermi energy ε_F as a function of the number of particles N?

c) [5 pts] What is the total energy E of the gas as a function of N at $T = 0$?

d) [5 pts] Give an estimate of the spatial extent $\bar{x}$ of the Fermi gas about the origin of the harmonic potential, at $T = 0$.

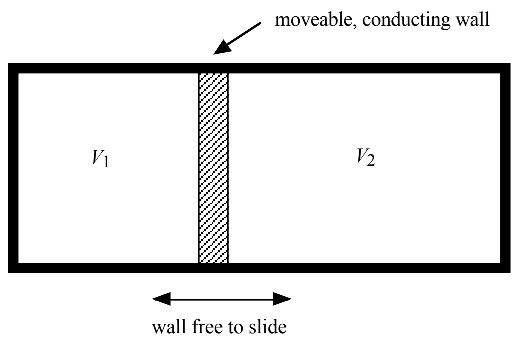
e) [5 pts] Consider now that the gas is in equilibrium at a fixed temperature T, and that $k_B T \gg \hbar\omega_0$, but one is still in the degenerate limit, $k_B T \ll \varepsilon_F$. As T increases, will the chemical potential μ increase, decrease, or remain the same? You must give a clear explanation for your answer.

f) [5 pts] For T as in the preceding part, estimate how the specific heat at constant volume C_V behaves as a function of temperature.

(turn over for problem 3)

3) [35 points total]

Consider a thermodynamic system consisting of two gases in volumes V_1 and V_2, separated by a thermally conducting, freely sliding wall, as shown in the diagram below. The gas in V_1 has N_1 particles, the gas in V_2 has N_2 particles, and particles cannot pass through the separating wall. The system is isolated from the rest of the universe, and the total volume $V_1 + V_2 = V$ is fixed. In thermal equilibrium, the pressures of the gases on the two sides of the sliding wall must be equal. The particles of each gas are non-relativisitic and indistinguishable.



If the system is initially in thermal equilibrium at temperature T, for each case below, explain in which direction the wall between the two gases will move if the temperature is increased a small amount ΔT. If you know what you are doing, you can get the answer with a simple graphical analysis, no algebra needed! But however you do it, you must give a sound explanation for your answer, not just a guess!

a) [10 pts] The gas in V_1 is an ideal gas of fermions in the degenerate limit, i.e. $k_B T \ll \epsilon_F$. The gas in V_2 is a classical ideal gas.

b) [12 pts] The gas in V_1 is an ideal gas of bosons in a Bose-Einstein condensed state, i.e. $T < T_c$. The gas in V_2 is a classical ideal gas.

c) [13 pts] The gas in V_1 is an ideal gas of fermions, and the gas in V_2 is an ideal gas of bosons. However both are now in the non-degenerate limit where their behavior is approaching that of an ideal classical gas (i.e. they obey the classical equation of state *with* the leading quantum correction).