PHY 418 **Midterm Exam** **Spring 2024**

For this exam you may use one 8.5"x11" sheet of paper on which you have written whatever notes you wish. Except for that sheet, this exam is closed book, closed notes, and you may not consult with any other person or resource in working out your solutions. Please write clearly. Please use a dark pen or pencil. The better you explain the steps you make in your solutions, the more likely it is that you can get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at.

Please write the academic honesty pledge, and sign your name, at the top of your work:
I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

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1) [30 points total]

(a) [10 pts] The following three equations are claimed to give the entropy S of various thermodynamic systems as a function of the total energy E, volume V, and number of particles N. However two of the three are inconsistent with one or more of the fundamental postulates of classical thermodynamics. Identify which are the inconsistent equations and explain why they are inconsistent. The quantities v_0, R, and θ are positive constants.

$$(i)\ S = \left(\frac{R}{v_0\theta}\right)^{1/3}(NVE)^{1/3}, \qquad (ii)\ S = \left(\frac{R}{v_0\theta}\right)\frac{V^3}{NE}, \qquad (iii)\ S = \left(\frac{R}{\theta^2}\right)^{1/3}\left(\frac{NE}{V}\right)^{2/3}$$

b) [20 pts] Consider the thermodynamics derivates below. For each of the five cases, use the Maxwell relations to find another thermodynamic derivative to which the given one is equal. Make sure to get your signs correct!

$$(i)\ \left(\frac{\partial S}{\partial p}\right)_{T,N} \qquad (ii)\ \left(\frac{\partial T}{\partial V}\right)_{S,N} \qquad (iii)\ \left(\frac{\partial N}{\partial V}\right)_{T,\mu} \qquad (iv)\ \left(\frac{\partial \mu}{\partial V}\right)_{T,N} \qquad (v)\ \left(\frac{\partial T}{\partial N}\right)_{S,p}$$

(turn over for problems 2 and 3)

2) [35 points total] You must explain your answer completely for all parts!

A box is partitioned by a wall into two parts, the right side and the left side. Each side is filled with an identical classical ideal gas of non-interacting, non-relativisitic, indistinguishable point particles of mass m. The gas on each side of the box has the same number of particles N. The volume of the left side of the box is V_1 and the volume of the right side of the box is V_2. The wall separating the two sides of the box is adiabatic (does not conduct heat), immoveable, and non-porous. Initially the gas on the left side is in equilibrium at temperature T_1, while the gas on the right side is in equilibrium at temperature T_2. The wall is now removed and each gas is free to fill the entire volume. The system then comes into its new state of equilibrium. In working out this problem you are free to use well known results about an ideal gas.

a) [5 pts] Is the final entropy of the total system larger or smaller than the initial total entropy?

b) [10 pts] What is the final temperature T_f of the gas?

c) [10 pts] What is the final total pressure p_f of the gas? Express p_f only in terms of T_1 and T_2 and the initial pressures of the two sides p_1 and p_2. If initially we had $T_1 = T_2$, is p_f greater or smaller than the average initial pressure $\bar{p} = (p_1 + p_2)/2$?

d) [10 pts] Compute the change in total entropy ΔS that results from removing the wall. Express your answer in terms of the variables T_1, T_2, V_1, V_2 and N only. Show that $\Delta S = 0$ if $V_1 = V_2$ and $T_1 = T_2$.

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3) [35 points total]

Consider a classical ideal gas of N non-interacting, non-relativistic, indistinguishable particles of mass m, moving in an external potential energy $U(\mathbf{r}) = \alpha z$ at a fixed temperature T (α is a fixed positive constant). The position of a particle $\mathbf{r} = (x, y, z)$ may lie anywhere in the range $0 \leq x \leq L$, $0 \leq y \leq L$, and $0 \leq z < \infty$. You may think of this as a model for a gas in a column of cross-sectional area L^2, in a gravitational field along the z-direction.

a) [10 pts] What is the probability density $\mathcal{P}(z)$ that a particle will be found at the height z?

b) [5 pts] What is the average height of a particle, i.e. what is $\langle z \rangle$?

c) [10 pts] What is the average total energy E of the gas?

d) [10 pts] What is the chemical potential μ of the gas, as a function of T, L, and N?