PHY 418 **Midterm Exam** **Spring 2025**

For this exam you may use one 8.5"x11" sheet of paper on which you have written whatever notes you wish. Except for that sheet, this exam is closed book, closed notes, and you may not consult with any other person or resource in working out your solutions. Please write clearly. Please use a dark pen or pencil. The better you explain the steps you make in your solutions, the more likely it is that you can get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at.

Please write the academic honesty pledge, and sign your name, at the top of your work:
I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

---

1) [20 points total]

For each thermodynamic derivative given below, find another thermodynamic derivative to which it is equal via a Maxwell relation. Remember to get your signs correct! [4 pts each part]

$$(i)\ \left(\frac{\partial T}{\partial N}\right)_{S,V}, \qquad (ii)\ \left(\frac{\partial \mu}{\partial p}\right)_{T,N}, \qquad (iii)\ \left(\frac{\partial N}{\partial T}\right)_{V,\mu}, \qquad (iv)\ \left(\frac{\partial T}{\partial p}\right)_{S,\mu}, \qquad (v)\ \left(\frac{\partial p}{\partial T}\right)_{V,N}$$

---

2 [40 points total]

Consider a classical ideal gas of N non-interacting, non-relativisitic, indistinguishable atoms of mass m, confined to a box of volume V and in equilibrium at temperature T. Each atom i has, in addition to its position $\mathbf{r}_i$ and its momentum $\mathbf{p}_i$, a net spin s_i which can be in one of three possible states, $s_i = -1, 0, +1$. The magnetic moment produced by this spin interacts with an external magnetic field $\mathbf{h} = h\hat{z}$ giving a contribution to the atom's energy, $\epsilon_i = -\mu h s_i$.

a) [15 pts] Find the canonical partition function $Q_N(T, V, h)$.

b) [8 pts] Find the pressure p as a function of T, V, N and h.

c) [8 pts] Find the average total energy $\langle E\rangle$ of the gas as a function of T, V, N and h.

d) [9 pts] Find the average total magnetization $\langle M\rangle = \mu\sum_{i=1}^{N}\langle s_i\rangle$ of the gas as a function of T, V, N and h.

(problem 3 is on the back)

3) [40 points total] Consider an ideal gas of non-relativistic, non-interacting, point particles, where the system contains two distinct types of particles, type A with mass m_A and type B with mass m_B. The particles of type A are indistinguishable from each other, the particles of type B are indistinguishable from each other, but particles of type A may be distinguished from particles of type B.

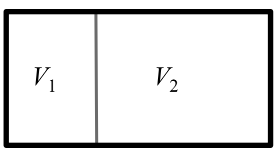
For a system of N_A particles of type A and N_B particles of type B, in equilibrium in a box of volume V at themperature T:

a) [10 pts] Find the Helmholtz free energy $A(T, V, N_A, N_B)$.

b) [5 pts] Find the pressure $p(T, V, N_A, N_B)$. Does the ideal gas law hold?

c) [5 pts] Find the chemical potentials $\mu_A(T, V, N_A, N_B)$ and $\mu_B(T, V, N_A, N_B)$ of the two types of particles.

Suppose now that the gas is in a box is that isolated from the surrounding environment (external walls are fixed, thermally insulating, and impermeable). Inside the box is a wall that divides the box into two sides 1 and 2, with volumes V_1 and V_2, such that the total volume is $V = V_1 + V_2$, as shown in the sketch below.



Initially the internal wall is thermally conducting, but immovable and impenetrable to particles. Initially the system is in equilibrium with $N_A^{(0)}$ particles of type A and no particles of type B in side 1, while there are $N_B^{(0)}$ particles of type B and no particles of type A in side 2. Because the internal wall is thermally conducting, the temperature on the two sides of the box are equal $T_1^{(0)} = T_2^{(0)} \equiv T$.

Now imagine that the wall suddenly becomes permeable to the passage of both types of particles (i.e. we open some small holes in the wall that allow particles to pass through). The wall remains thermally conducting but immovable. Particles flow across the wall until the system comes to its new equilibrium.

d) [5 pts] Does the temperature of the gas on either side of the wall change? Explain why or why not.

e) [10 pts] What thermodynamic condition will determine the number of particles N_{A1} and N_{B1} that are on side 1, and the number of particles N_{A2} and N_{B2} that are on side 2, when the system reaches equilibrium? Apply that condition to determine N_{A1}, N_{B1}, N_{A2}, and N_{B2}.

f) [5 pts] What now are the pressures p_1 and p_2 on each side of the box? Have the pressures changed from what they were initially (when the wall was impermeable).