PHYS 418 Solutions Problem Set 8

1)

a) We want $\langle \vec{r}_1, \vec{r}_2, \ell, m_z \mid \vec{k}_1, \vec{k}_2, \ell', m_z' \rangle$

If we did not have to worry about symmetry, this would just be

$$\frac{1}{V} e^{i(\vec{k}_1\cdot\vec{r}_1 + \vec{k}_2\cdot\vec{r}_2)} \delta_{\ell\ell'} \delta_{m_z m_z'}$$

But we must have that the total wave function is antisymmetric under exchange of the two particles $(\vec{r}_1, s_1) \leftrightarrow (\vec{r}_2, s_2)$.

Now we know from their definition that the triplet states are symmetric in interchange of the two spins ie $|\ell=1, m_z\rangle \to +|\ell=1, m_z\rangle$ as $s_1 \leftrightarrow s_2$.

But the singlet state is antisymmetric ie $|\ell=0,0\rangle \to -|\ell=0,0\rangle$ as $s_1 \leftrightarrow s_2$

Therefore the spatial part of the wave function must be antisymmetric for the three triplet spin states, but it must be symmetric for the spin singlet state

triplet: $\langle \vec{r}_1, \vec{r}_2, \ell=1, m_z \mid \vec{k}_1, \vec{k}_2, \ell', m_z' \rangle = \frac{1}{\sqrt{2}V}\left[e^{i(\vec{k}_1\cdot\vec{r}_1 + \vec{k}_2\cdot\vec{r}_2)} - e^{i(\vec{k}_2\cdot\vec{r}_1 + \vec{k}_1\cdot\vec{r}_2)}\right]\delta_{\ell\ell'}\delta_{m_z m_z'}$

singlet: $\langle \vec{r}_1, \vec{r}_2, \ell=0, m_z \mid \vec{k}_1, \vec{k}_2, \ell', m_z' \rangle = \frac{1}{\sqrt{2}V}\left[e^{i(\vec{k}_1\cdot\vec{r}_1 + \vec{k}_2\cdot\vec{r}_2)} + e^{i(\vec{k}_2\cdot\vec{r}_1 + \vec{k}_1\cdot\vec{r}_2)}\right]\delta_{\ell\ell'}\delta_{m_z m_z'}$

b) So $\langle \vec{r}_1, \vec{r}_2, \ell, m_z | \hat{\rho}_2 | \vec{r}_1, \vec{r}_2, \ell, m_z \rangle$

$$= \frac{1}{Q_2} \sum_{|\vec{k}_1, \vec{k}_2, \ell', m_z'\rangle} e^{-\frac{\beta \hbar^2}{2m}(k_1^2 + k_2^2)} \left| \langle \vec{r}_1, \vec{r}_2, \ell, m_z | \vec{k}_1, \vec{k}_2, \ell', m_z' \rangle \right|^2$$

$$= \frac{1}{Q_2} \frac{1}{2} \sum_{\vec{k}_1, \vec{k}_2} e^{-\frac{\beta \hbar^2}{2m}(k_1^2 + k_2^2)} \frac{\left| e^{i(\vec{k}_1 \cdot \vec{r}_1 + \vec{k}_2 \cdot \vec{r}_2)} \pm e^{i(\vec{k}_2 \cdot \vec{r}_1 + \vec{k}_1 \cdot \vec{r}_2)} \right|^2}{2V^2}$$

where (+) is for the singlet state $|\ell, m_z\rangle = |0, 0\rangle$
and (−) is for the triplet states $|\ell, m_z\rangle = |1, 1\rangle, |1, 0\rangle, |1, -1\rangle$

But the above is exactly the same as we had in lecture for the spinless particles, with (+) for the spinless bosons, and (−) for the spinless fermions. The only difference is in the evaluation of Q_2, where we now have to sum over the different spin states. However Q_2 is just a constant and does not effect the dependence on $|\vec{r}_1 - \vec{r}_2|$.

Therefore, two spin-½ fermions in a spin triplet state with $\ell = 1$ behave like spinless fermions with a short range repulsion, while two spin-½ fermions in a spin singlet state with $\ell = 0$ behave like spinless bosons with a short range attraction. This is reasonable since the opposite oriented spins of the singlet state take care of the required antisymmetry of the wavefunction, so that the spatial dependence is symmetric, just like with bosons.

It is interesting to also consider the matrix elements of $\hat{\rho}_2$ for two fermions in the spin basis states of $|s_1\rangle|s_2\rangle$

If we want $\langle \vec{r}_1, \vec{r}_2, s_1=\uparrow, s_2=\uparrow | \hat{\rho}_2 | \vec{r}_1, \vec{r}_2, s_1=\uparrow, s_2=\uparrow \rangle$

then since $|\uparrow\rangle|\uparrow\rangle = |\ell=1, m_z=1\rangle$ then this will behave just like the two spinless fermions.

But now consider if the fermion spins are opposite

$\langle \vec{r}_1, \vec{r}_2, s_1=\uparrow, s_2=\downarrow | \hat{\rho}_2 | \vec{r}_1, \vec{r}_2, s_1=\uparrow, s_2=\downarrow \rangle$

Now we take the energy eigenstates as $|\vec{k}_1, \vec{k}_2, \sigma_1, \sigma_2\rangle$ and

$$\langle \vec{r}_1, \vec{r}_2, s_1, s_2 | \vec{k}_1, \vec{k}_2, \sigma_1, \sigma_2 \rangle = e^{i(\vec{k}_1\cdot\vec{r}_1 + \vec{k}_2\cdot\vec{r}_2)} \delta_{s_1\sigma_1}\delta_{s_2\sigma_2} - e^{i(\vec{k}_2\cdot\vec{r}_1 + \vec{k}_1\cdot\vec{r}_2)} \delta_{s_2\sigma_1}\delta_{s_1\sigma_2}$$

where the $(-)$ sign is because this must be antisymmetric under exchange of the two fermions $\vec{r}_1, s_1 \longleftrightarrow \vec{r}_2, s_2$

Then

$$\langle \vec{r}_1, \vec{r}_2, \uparrow, \downarrow | \hat{\rho}_2 | \vec{r}_1, \vec{r}_2, \uparrow, \downarrow \rangle =$$

$$\frac{1}{Q_2} \sum_{|\vec{k}_1, \vec{k}_2, \sigma_1, \sigma_2\rangle} e^{-\frac{\beta\hbar^2 k^2}{2m}} \left| e^{i(\vec{k}_1\cdot\vec{r}_1 + \vec{k}_2\cdot\vec{r}_2)} \delta_{\uparrow\sigma_1}\delta_{\downarrow\sigma_2} - e^{i(\vec{k}_2\cdot\vec{r}_1 + \vec{k}_1\cdot\vec{r}_2)} \delta_{\downarrow\sigma_1}\delta_{\uparrow\sigma_2} \right|^2$$

The only terms in this sum which are not zero are when
(1) $\sigma_1 = \uparrow$ and $\sigma_2 = \downarrow$ OR when (2) $\sigma_2 = \uparrow$ and $\sigma_1 = \downarrow$

If ① then only the first term $\sim \delta_{\uparrow\sigma_1}\delta_{\downarrow\sigma_2} \neq 0$ and the second term vanishes. If ② then only the second term $\sim \delta_{\downarrow\sigma_1}\delta_{\uparrow\sigma_2} \neq 0$ and the first term vanishes.

In either case we have only one term present, so the absolute value squared is $\left|e^{i(\bar{k}_1\cdot\bar{r}_1 + \bar{k}_2\cdot\bar{r}_2)}\right|^2 = \left|e^{i(\bar{k}_2\cdot\bar{r}_1 + \bar{k}_1\cdot\bar{r}_2)}\right|^2 = 1$

So $\langle \bar{r}_1, \bar{r}_2, \uparrow, \downarrow | \hat{\rho}_2 | \bar{r}_1, \bar{r}_2, \uparrow, \downarrow \rangle$ is <u>constant</u> independent of $|\bar{r}_1 - \bar{r}_2|$, just like is the case for <u>classical</u> particles

what is the reason for this?

$\langle \bar{r}_1, \bar{r}_2, s_1 = \uparrow, s_2 = \downarrow | \hat{\rho}_2 | \bar{r}_1, \bar{r}_2, s_1 = \uparrow, s_2 = \downarrow \rangle$ is just the probability that the "up" particle is at $\bar{r}_1$, while the "down" particle is at $\bar{r}_2$. Putting it that way, we see that we are using spin to <u>distinguish</u> between the two particles, hence particles are no longer indistinguishable, so it is no surprise that the quantum interference effects vanish!

That is different from the matrix element with respect to $|l=0, m_z=0\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right]$ because in that case we are not specifying which particle is up and which is down, only that their spins are opposite.

2)

a) For a quantum ideal gas, prob to have occupations $\{n_i\}$:

$$P(\{n_i\}) = \frac{e^{-\beta \sum_i (\epsilon_i - \mu) n_i}}{\mathcal{L}}$$

where $\mathcal{L} = \prod_j \left[\sum_n e^{-\beta(\epsilon_j - \mu) n} \right]$

$$\Rightarrow P(\{n_i\}) = \frac{\prod_i \left[e^{-\beta(\epsilon_i - \mu) n_i} \right]}{\prod_j \left[\sum_n e^{-\beta(\epsilon_j - \mu) n} \right]}$$

$$= \prod_i \left[\frac{e^{-\beta(\epsilon_i - \mu) n_i}}{\sum_n e^{-\beta(\epsilon_i - \mu) n}} \right].$$

$$\boxed{P(\{n_i\}) = \prod_i p_i(n_i)}$$

$p_i(n_i)$ = prob there are n_i particles in state i

b)
$$S = -k_B \sum_{\{n_i\}} P(\{n_i\}) \ln P(\{n_i\})$$

$$= -k_B \sum_{\{n_i\}} \prod_i p_i(n_i) \ln \left[\prod_j p_j(n_j) \right]$$

$$= -k_B \sum_{\{n_i\}} \left[\prod_i p_i(n_i) \right] \sum_j \ln p_j(n_j)$$

$$S = -k_B \sum_{n_1} \sum_{n_2} \cdots \Big\{ (p_1(n_1) p_2(n_2) \cdots) \ln p_1(n_1)$$
$$+ (p_1(n_1) p_2(n_2) \cdots) \ln p_2(n_2)$$
$$+ \vdots \Big\}$$

In each term we can do all the $\sum_{n_k}$ except for the p_k that is in the logarithm, because

$$\sum_k p_k(n_k) = 1 \quad \text{(check from def of } p_k(n_k)\text{)}$$

ie for example $\sum_{n_1} \sum_{n_2} \sum_{n_3} p_1(n_1) p_2(n_2) p_3(n_3) \ln p_1(n_1)$

$$= \sum_{n_1} p_1(n_1) \ln p_1(n_1) \Big(\sum_{n_2} p_2(n_2)\Big)\Big(\sum_{n_3} p_3(n_3)\Big)$$
$$= \sum_{n_1} p_1(n_1) \ln p_1(n_1)$$

So

$$S = -k_B \Big\{ \sum_{n_1} p_1(n_1) \ln p(n_1) + \sum_{n_2} p_2(n_2) \ln p(n_2) + \cdots$$

$$\boxed{S = -k \sum_i \sum_{n_i} p_i(n_i) \ln p_i(n_i)}$$

c) Fermions: $n = 0$ or 1 only

$$p(1) = \frac{e^{-\beta(\epsilon-\mu)}}{1+e^{-\beta(\epsilon-\mu)}} = \frac{1}{e^{\beta(\epsilon-\mu)}+1} = \langle n \rangle$$

$$p(0) = 1 - p(1) = 1 - \langle n \rangle$$

$$\sum_n p(n) \ln p(n) = p(0) \ln p(0) + p(1) \ln p(1)$$

$$= (1-\langle n \rangle) \ln (1-\langle n \rangle) + \langle n \rangle \ln \langle n \rangle$$

$$S = -k_B \sum_i \sum_n p_i(n_i) \ln p_i(n_i)$$

$$\boxed{S = k_B \sum_i \left\{ -(1-\langle n_i \rangle) \ln (1-\langle n_i \rangle) - \langle n_i \rangle \ln \langle n_i \rangle \right\}}$$

Bosons: $n = 0, 1, 2 \ldots$

$$p(n) = \frac{e^{-\beta(\epsilon-\mu)n}}{\sum_n e^{-\beta(\epsilon-\mu)n}} \equiv \frac{e^{-\beta(\epsilon-\mu)n}}{w}$$

$$\sum_n p(n) \ln p(n) = \sum_n \left(\frac{e^{-\beta(\epsilon-\mu)n}}{w} \right) \ln \left(\frac{e^{-\beta(\epsilon-\mu)n}}{w} \right)$$

$$= \sum_n \left(\frac{e^{-\beta(\epsilon-\mu)n}}{w} \right) \left\{ -\beta(\epsilon-\mu)n - \ln w \right\}$$

$$= -\beta(\epsilon-\mu) \frac{\sum_n e^{-\beta(\epsilon-\mu)n} n}{w} - (\ln w) \frac{\sum_n e^{-\beta(\epsilon-\mu)n}}{w}$$

$$= -\beta(\epsilon-\mu) \langle n \rangle - \ln w$$

now $w = \sum_{n=0}^{\infty} e^{-\beta(\epsilon-\mu)n} = \dfrac{1}{1-e^{-\beta(\epsilon-\mu)}}$

$$= \frac{e^{\beta(\epsilon-\mu)}}{e^{\beta(\epsilon-\mu)}-1} = 1 + \frac{1}{e^{\beta(\epsilon-\mu)}-1} = 1+\langle n\rangle$$

So

$$\sum_n p(n)\ln p(n) = -\beta(\epsilon-\mu)\langle n\rangle - \ln(1+\langle n\rangle)$$

$$= -\Big\{\beta(\epsilon-\mu)\langle n\rangle + \ln(1+\langle n\rangle) + \langle n\rangle\ln(1+\langle n\rangle) - \langle n\rangle \ln(1+\langle n\rangle)\Big\}$$

$$= -\Big\{(1+\langle n\rangle)\ln(1+\langle n\rangle) + \langle n\rangle \ln\left[\frac{e^{\beta(\epsilon-\mu)}}{1+\langle n\rangle}\right]\Big\}$$

$$= -\Big\{(1+\langle n\rangle)\ln(1+\langle n\rangle) + \langle n\rangle \ln\left[e^{\beta(\epsilon-\mu)}\left(1-e^{-\beta(\epsilon-\mu)}\right)\right]\Big\}$$

$$= -\Big\{(1+\langle n\rangle)\ln(1+\langle n\rangle) + \langle n\rangle \ln\left(e^{\beta(\epsilon-\mu)}-1\right)\Big\}$$

$$= -\Big\{(1+\langle n\rangle)\ln(1+\langle n\rangle) - \langle n\rangle \ln\langle n\rangle\Big\}$$

$$S = -k_B \sum_i \sum_{n_i} p_i(n_i)\ln p_i(n_i)$$

$$\boxed{S = k_B \sum_i \Big\{(1+\langle n_i\rangle)\ln(1+\langle n_i\rangle) - \langle n_i\rangle \ln\langle n_i\rangle\Big\}}$$

3) Let us define $\Delta \equiv \beta(\varepsilon_i - \mu)$. Then

$$p(n_i) = \frac{e^{-\Delta n_i}}{\sum_n e^{-\Delta n}}$$

a) $$\langle n_i \rangle = \sum_{n_i} p(n_i) n_i = \frac{\sum_{n_i} n_i e^{-\Delta n_i}}{\sum_n e^{-\Delta n}}$$

Note $$\sum_{n_i} n_i e^{-\Delta n_i} = -\frac{\partial}{\partial \Delta}\left[\sum_{n_i} e^{-\Delta n_i}\right]$$

So if we define $g \equiv \sum_n e^{-\Delta n}$ then

$$\langle n_i \rangle = \frac{-\frac{\partial g}{\partial \Delta}}{g} = -\frac{\partial \ln g}{\partial \Delta}$$

<u>Bosons</u> $n = 0, 1, 2, \ldots$

$$g = \sum_{n=0}^{\infty} e^{-\Delta n} = \frac{1}{1 - e^{-\Delta}}$$ geometric series

$$\frac{\partial g}{\partial \Delta} = \frac{-e^{-\Delta}}{(1 - e^{-\Delta})^2}$$ (keep track of the minus signs!)

$$\langle n_i \rangle = \frac{-\frac{\partial g}{\partial \Delta}}{g} = \frac{e^{-\Delta}}{(1 - e^{-\Delta})^2}(1 - e^{-\Delta}) = \frac{e^{-\Delta}}{1 - e^{-\Delta}}$$

$$= \frac{1}{e^{\Delta} - 1}$$

substitute in for $\Delta = \beta(\varepsilon_i - \mu)$ to get

Bosons

$$\boxed{\langle n_i \rangle = \frac{1}{e^{\beta(\varepsilon_i - \mu)} - 1}}$$

the familiar Bose occupation function

Fermions

$n = 0, 1$ only

$$q = \sum_{n=0}^{1} e^{-\Delta n} = 1 + e^{-\Delta}$$

$$\frac{\partial q}{\partial \Delta} = -e^{-\Delta}$$

$$\langle n_i \rangle = \frac{-\frac{\partial q}{\partial \Delta}}{q} = \frac{e^{-\Delta}}{1+e^{-\Delta}} = \frac{1}{e^{\Delta}+1}$$

substitute in for $\Delta = \beta(\varepsilon_i - \mu)$ to get

Fermions

$$\boxed{\langle n_i \rangle = \frac{1}{e^{\beta(\varepsilon_i - \mu)} + 1}}$$

the familiar Fermi occupation function.

b) Now we compute $\langle n_i^2 \rangle$

$$\langle n_i^2 \rangle = \sum_{n_i} p(n_i) n_i^2 = \frac{\sum_{n_i} n_i^2 e^{-\Delta n_i}}{\sum_n e^{-\Delta n}}$$

Note: $$\sum_{n_i} n_i^2 e^{-\Delta n_i} = \frac{\partial^2}{\partial \Delta^2}\left[\sum_{n_i} e^{-\Delta n_i}\right] = \frac{\partial^2 g}{\partial \Delta^2}$$

<u>Bosons:</u> we had from (a) $$\frac{\partial g}{\partial \Delta} = \frac{-e^{-\Delta}}{(1-e^{-\Delta})^2}$$

$$\Rightarrow \frac{\partial^2 g}{\partial \Delta^2} = \frac{e^{-\Delta}}{(1-e^{-\Delta})^2} + \frac{2e^{-2\Delta}}{(1-e^{-\Delta})^3}$$

$$\langle n_i^2 \rangle = \frac{\frac{\partial^2 g}{\partial \Delta^2}}{g} = \left[\frac{e^{-\Delta}}{(1-e^{-\Delta})^2} + \frac{2e^{-2\Delta}}{(1-e^{-\Delta})^3}\right](1-e^{-\Delta})$$

$$= \frac{e^{-\Delta}}{1-e^{-\Delta}} + \frac{2e^{-2\Delta}}{(1-e^{-\Delta})^2}$$

$$= \frac{1}{e^{\Delta}-1} + \frac{2}{(e^{\Delta}-1)^2}$$

$$= \langle n_i \rangle + 2\langle n_i \rangle^2$$

So fluctuation is

$$\langle n_i^2 \rangle - \langle n_i \rangle^2 = \langle n_i \rangle + 2\langle n_i \rangle^2 - \langle n_i \rangle^2$$
$$= \langle n_i \rangle + \langle n_i \rangle^2$$
$$= \langle n_i \rangle (1 + \langle n_i \rangle)$$

$$\langle n_i^2\rangle - \langle n_i\rangle^2 = \langle n_i\rangle\left(1+\langle n_i\rangle\right)$$

$$= \frac{1}{e^\Delta-1}\left(1+\frac{1}{e^\Delta-1}\right) = \frac{1}{e^\Delta-1}\;\frac{e^\Delta}{e^\Delta-1}$$

$$= \frac{e^\Delta}{(e^\Delta-1)^2} = \frac{e^{\beta(\varepsilon_i-\mu)}}{\left[e^{\beta(\varepsilon_i-\mu)}-1\right]^2} = e^{\beta(\varepsilon_i-\mu)}\langle n_i\rangle^2$$

Bosons:

$$\boxed{\begin{aligned}\langle n_i^2\rangle - \langle n_i\rangle^2 &= \langle n_i\rangle\left(1+\langle n_i\rangle\right)\\ &= e^{\beta(\varepsilon_i-\mu)}\langle n_i\rangle^2\\ &= \frac{e^{\beta(\varepsilon_i-\mu)}}{\left[e^{\beta(\varepsilon_i-\mu)}-1\right]^2}\end{aligned}}$$

<u>Fermions</u> from (a) $\frac{\partial g}{\partial\Delta} = -e^{-\Delta}$

$$\Rightarrow \frac{\partial^2 g}{\partial\Delta^2} = e^{-\Delta}$$

$$\langle n_i^2\rangle = \frac{\frac{\partial^2 g}{\partial\Delta^2}}{g} = \frac{e^{-\Delta}}{1+e^{-\Delta}} = \frac{1}{e^\Delta+1} = \langle n_i\rangle$$

So $\langle n_i^2\rangle - \langle n_i\rangle^2 = \langle n_i\rangle - \langle n_i\rangle^2 = \langle n_i\rangle\left(1-\langle n_i\rangle\right)$

$$\langle n_i^2\rangle - \langle n_i\rangle^2 = \langle n_i\rangle\left(1 - \langle n_i\rangle\right)$$

$$= \frac{1}{e^{\Delta}+1}\left(1 - \frac{1}{e^{\Delta}+1}\right)$$

$$= \frac{1}{e^{\Delta}+1}\,\frac{e^{\Delta}}{e^{\Delta}+1} = \frac{e^{\Delta}}{(e^{\Delta}+1)^2}$$

$$= e^{\Delta}\langle n_i\rangle^2$$ same as for bosons!

Fermions

$$\boxed{\begin{aligned} \langle n_i^2\rangle - \langle n_i\rangle^2 &= \langle n_i\rangle\left(1 - \langle n_i\rangle\right) \\ &= e^{\beta(\varepsilon_i-\mu)}\langle n_i\rangle^2 \\ &= \frac{e^{\beta(\varepsilon_i-\mu)}}{\left[e^{\beta(\varepsilon_i-\mu)}+1\right]^2} \end{aligned}}$$

c)

$$N = \sum_i n_i \quad \Rightarrow \quad \langle N\rangle = \sum_i \langle n_i\rangle$$

$$\langle N\rangle^2 = \left(\sum_i \langle n_i\rangle\right)\left(\sum_j \langle n_j\rangle\right) = \sum_{i,j} \langle n_i\rangle\langle n_j\rangle$$

$$= \sum_i \langle n_i\rangle^2 + \sum_{i\neq j} \langle n_i\rangle\langle n_j\rangle$$

$$\langle N^2\rangle = \left\langle \sum_i n_i \sum_j n_j \right\rangle = \sum_{i,j} \langle n_i n_j\rangle$$

$$= \sum_i \langle n_i^2\rangle + \sum_{i\neq j} \langle n_i n_j\rangle$$

But n_i and n_j are statistically independent
$\Rightarrow \langle n_i n_j \rangle = \langle n_i \rangle \langle n_j \rangle$ when $i \neq j$

$$\Rightarrow \langle N^2 \rangle = \sum_i \langle n_i^2 \rangle + \sum_{i \neq j} \langle n_i \rangle \langle n_j \rangle$$

and so

$$\langle N^2 \rangle - \langle N \rangle^2 = \sum_i \langle n_i^2 \rangle + \sum_{i \neq j} \langle n_i \rangle \langle n_j \rangle$$
$$- \sum_i \langle n_i \rangle^2 - \sum_{i \neq j} \langle n_i \rangle \langle n_j \rangle$$

$$\boxed{\langle N^2 \rangle - \langle N \rangle^2 = \sum_i \left[\langle n_i^2 \rangle - \langle n_i \rangle^2 \right]}$$

For Bosons

$$\langle N^2 \rangle - \langle N \rangle^2 = \sum_i \left(\langle n_i \rangle + \langle n_i \rangle^2 \right)$$
$$= \langle N \rangle + \sum_i \langle n_i \rangle^2$$

For Fermions

$$\langle N^2 \rangle - \langle N \rangle^2 = \sum_i \left[\langle n_i \rangle - \langle n_i \rangle^2 \right]$$
$$= \langle N \rangle - \sum_i \langle n_i \rangle^2$$

For a classical ideal gas,

$$\langle N^2 \rangle - \langle N \rangle^2 = \langle N \rangle$$

(follows from problem 1 on Set 6, and the properties of the Poisson distribution)

Since $\sum_i \langle n_i \rangle^2 > 0$ we conclude:

$$\boxed{\begin{array}{l}\text{fluctuations in } N \text{ are \underline{larger} for an ideal Bose gas than for a classical gas} \\ \text{fluctuation in } N \text{ are \underline{smaller} for an ideal Fermi gas than for a classical gas}\end{array}}$$