PHYS 418 **Endterm Exam** **Spring 2026**

For this exam you may use one 8.5"x11" sheet of paper on which you have written notes on one side of the page. No other books, notes, or resources are permitted. Please write clearly with a dark pen or pencil. The better you explain the steps you make in your solutions, the more likely it is that you can get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at.

Please write the academic honesty pledge, and **sign** your name, at the top of your work:
I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Do any **TWO** of the three following problems. If you do all three, your grade will be determined from your performance on the best two.

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1) [50 points total]

Consider atoms A that can bind together to form a diatomic molecule A_2,

$$2A \quad \leftrightarrow \quad A_2$$

The binding energy of the molecule is Δ (i.e. the ground state of the molecule A_2 has energy $-\Delta$ compared to the ground state of two unbound atoms). Assume that the atoms and diatomic molecules can be treated as non-interacting, non-relativistic, indistinguishable, classical point particles (i.e. ignore any rotational, vibrational, or electronic excitations). Suppose that there are initially N atoms A and no molecules A_2, confined to a cubic box of volume V at temperature T. What will be the ratio of the number of atoms A to the number of molecules A_2 when the system comes into thermal and chemical equilibrium? You may assume that the mass of A_2 is twice the mass of A.

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2) [50 points total]

Consider a gas of N indistinguishable, non-relativistic, non-interacting spin 1/2 fermions (spin degeneracy $g_s = 2$), confined to move in a one dimensional system of length L. Assume periodic boundary conditions.

a) [10 pts] The density of states $g(\varepsilon)$ is defined such that $g(\varepsilon)d\epsilon$ is the number of single-particle energy levels with energy between ε and $\varepsilon + d\varepsilon$, per unit energy per unit length. Compute $g(\varepsilon)$ for this fermi gas.

b) [10 pts] Compute the Fermi energy of the gas as a function of its density $n = N/L$.

c) [10 pts] Compute the pressure of the gas at $T = 0$ as a function of its density n.

d) [10 pts] *Estimate* the value of c_V, the specific heat at constant volume, per unit volume, for low temperatures $T \ll T_F$.

problem 2 is continued on back side

2e) [10 pts] Consider a rectangular shaped conducting wire where the length in one direction L_z is extremely large (you may take $L_z \to \infty$), but the lengths in the two orthogonal directions $L_x = L_y \equiv L_\perp$ are finite and very small compared to L_z. Assuming periodic boundary conditions in all directions, the single-particle energy eigenstates in such a wire may be labeled by a wavevector $\mathbf{k}$, with $k_z = (2\pi/L_z)m_z$, $k_x = (2\pi/L_\perp)m_x$, and $k_y = (2\pi/L_\perp)m_y$, with $m_x, m_y, m_z = 0, \pm 1, \pm 2, \ldots$ integer. The gas of conduction electrons in this wire can be considered one dimensional if all the electrons occupy single particle states with $m_x = m_y = 0$. For a given density $n = N/L_z$ of electrons per unit length, how small does $L_\perp$ have to be for the gas to be considered one dimensional at $T = 0$?

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3) [50 points total]

Consider an ideal Bose gas of indistinguishable, spinless, free, non-relativistic particles in a box in 3 dimensions. Each particle has an internal degree of freedom that can take only one of two energy values, the ground state with $\varepsilon_0 = 0$ and an excited state with $\varepsilon_1 > 0$.

a) [10 pts] If the Bose-Einstein transition temperature of the Bose gas when $\varepsilon_1 \to \infty$ (this is just the B-E transition temperature of point particles) is T_{c0}, do you expect the B-E transition temperature T_c of the gas with finite ε_1 will be larger or smaller than T_{c0}? No calculation is needed, just a clear explanation.

b) [20 pts] Write down an equation from which the value of T_c could in principle be determined. For full credit on this part, you should express your results in terms of the *standard Bose function* $g_{3/2}(z)$ and the thermal wavelength λ.

c) [20 pts] Now assume that $\varepsilon_1/k_BT \gg 1$ for all temperatures of interest. Find the relative shift in the B-E transition temperature, $(T_c - T_{c0})/T_{c0}$, to lowest non-zero order.

You might find the following helpful:

The density of states for spinless, free, non-relativistic point particles in 3 dimensions is:

$$g(\varepsilon) = \frac{2}{\sqrt{\pi}\lambda^3} \frac{1}{k_BT} \sqrt{\frac{\varepsilon}{k_BT}}$$

The standard Bose functions are:

$$g_n(z) \equiv \frac{1}{\Gamma(n)} \int_0^\infty dy \frac{y^{n-1}}{z^{-1}e^y - 1} = \sum_{\ell=1}^\infty \frac{z^\ell}{\ell^n} \quad \text{where} \quad \Gamma(n+1) = n\Gamma(n) \text{ and } \Gamma(1/2) = \sqrt{\pi}$$

The thermal wavelength is:

$$\lambda = \sqrt{\frac{h^2}{2\pi m k_BT}} \tag{1}$$

Note: do not confuse the density of states $g(\varepsilon)$ with the standard Bose function $g_n(z)$. Even though they both use the letter "g", they are two different things!