PHY 418 **Midterm Exam** **Spring 2026**

For this exam you may use one 8.5"x11" sheet of paper on which you have written whatever notes you wish. Except for that sheet, this exam is closed book, closed notes, and you may not consult with any other person or resource in working out your solutions. Please write clearly. Please use a dark pen or pencil. The better you explain the steps you make in your solutions, the more likely it is that you can get partial credit if you have done something incorrectly. Please put a box around your final answer to each question. Cross out anything you don't want me to look at.

Please write the academic honesty pledge, and sign your name, at the top of your work: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

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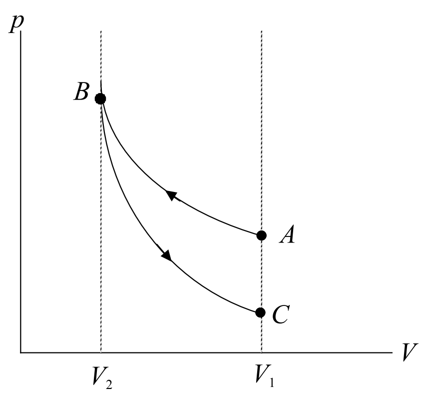
1) [30 points total]

a) [15 pts] Consider the thermodynamic derivatives below. For each of the five cases, use the Maxwell relations to find another thermodynamic derivative to which the given one is equal. Make sure to get your signs correct!

$$(i)\ \left(\frac{\partial V}{\partial T}\right)_{p,N} \qquad (ii)\ \left(\frac{\partial T}{\partial V}\right)_{S,N} \qquad (iii)\ \left(\frac{\partial p}{\partial \mu}\right)_{T,V} \qquad (iv)\ \left(\frac{\partial \mu}{\partial p}\right)_{S,N} \qquad (v)\ \left(\frac{\partial \mu}{\partial T}\right)_{V,N}$$

b) [15 pts] Consider a classical ideal gas of N particles, that is compressed and then expanded, as shown in the diagram below. The gas starts at point A in the p-V diagram, with volume V_1 at temperature T_1. It is then *isothermally* compressed (i.e. at constant temperature) to volume V_2, denoted as point B on the diagram. The gas is then *adiabatically* expanded (i.e. at constant entropy) back to volume V_1, denoted point C on the diagram. What is the final temperature T_2 at point C? Express your answer in terms of T_1, V_1, and V_2.



It might help you to know that the Sackur-Tetrode equation for the entropy S of an ideal gas is,

$$S(E,V,N) = \frac{5}{2}Nk_B + Nk_B \ln\left[\frac{V}{h^3 N}\left(\frac{4\pi m E}{3N}\right)^{3/2}\right]$$

2) [35 points total] You must explain your answer completely for all parts!

A box is partitioned by a wall into two parts, the right side and the left side. The exterior walls of the box are thermally insulating. Each side is filled with a classical ideal gas of non-interacting, non-relativisitic, indistinguishable point particles of mass m. The gases on each side of the box are the same type of gas, and each has the same number of particles N. The volume of the left side of the box is V_1 and the volume of the right side of the box is V_2. The wall separating the two sides of the box is thermally insulating, immoveable, and non-porous. Initially the gas on the left side is in equilibrium at temperature T_1, while the gas on the right side is in equilibrium at temperature T_2. The wall is now removed and each gas is free to fill the entire volume. The system then comes into its new state of equilibrium. In working out this problem you are free to use well known results about an ideal gas.

a) [5 pts] Is the final entropy of the total system larger or smaller than the initial total entropy?

b) [10 pts] What is the final temperature T_f of the gas?

c) [10 pts] What is the final total pressure p_f of the gas? Express p_f only in terms of T_1 and T_2 and the initial pressures of the two sides p_1 and p_2. If initially we had $T_1 = T_2$, is p_f greater or smaller than the average initial pressure $\bar{p} = (p_1 + p_2)/2$?

d) [10 pts] Compute the change in total entropy ΔS that results from removing the wall. Express your answer in terms of the variables T_1, T_2, V_1, V_2 and N only. Show that $\Delta S = 0$ if $V_1 = V_2$ and $T_1 = T_2$.

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3) [35 points total]

Consider an ideal gas of N non-relativistic, non-interacting, indistinguishable point particles, constrained to move in one dimension, given by the coordinate x. The value of x is unbounded, i.e. $x \in (-\infty, +\infty)$. The particles move in an external potential energy given by $U(x) = \alpha|x|$.

a) [7 pts] Compute the N-particle partition function $Q_N(T)$ for this gas.

b) [7 pts] Find the entropy of this gas $S(T, N)$.

c) [7 pts] Find the specific heat of this gas C.

d) [7 pts] Find the root mean square displacement of particles from the origin $x_{rms} \equiv \sqrt{\langle x^2 \rangle}$.

e) [7 pts] What is the probability that a particle is found at a position x, such that $|x| > x_{rms}$?