Motion in Uniform Magnetic field

$$\dot{\vec{r}} = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial \vec{k}} \qquad \hbar\dot{\vec{k}} = -e\frac{1}{c}\vec{v}(\vec{k})\times\vec{H}$$

For motion in uniform field, $\dot{\varepsilon}(\vec{k}(t)) = \frac{d\varepsilon}{d\vec{k}}\cdot\frac{d\vec{k}}{dt} = \hbar\vec{v}\cdot\dot{\vec{k}} = 0$

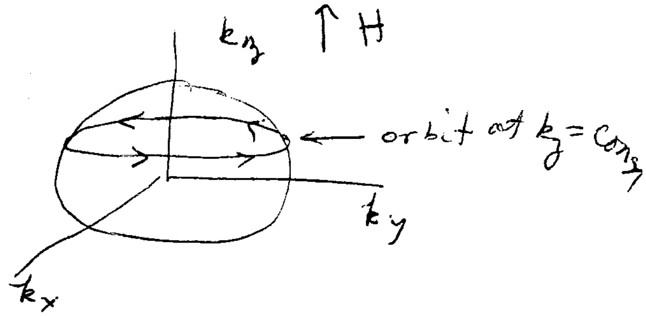
since $\vec{v}\cdot(\vec{v}\times\vec{H}) = 0$

so electron moves on surface of constant energy.

also $\frac{d}{dt}(\vec{k}\cdot\vec{H}) = \dot{\vec{k}}\cdot\vec{H} = 0$ as $\vec{H}\cdot(\vec{v}\times\vec{H}) = 0$

⇒ electrons move on curves formed by intersection of plane of constant ~~k_z (take H in z)~~ $k_\parallel$, with surfaces of constant energy.

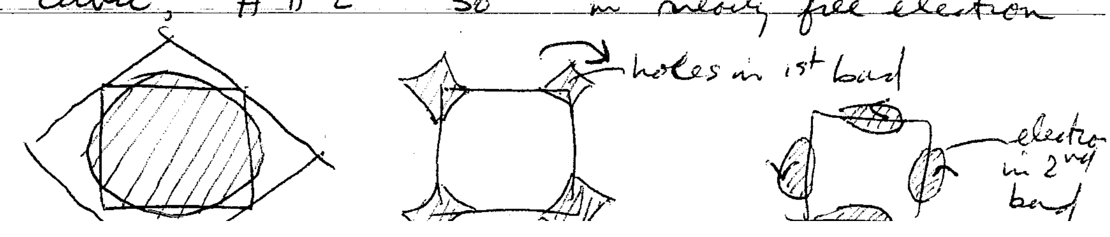
For spherical energy surface



Sense of orbit: since $\vec{v} = \frac{1}{\hbar}\frac{\partial\varepsilon}{\partial\vec{k}}$ points from low ε to higher ε.
If H is up, one walks in orbit so that higher energy states are on right as $\dot{\vec{k}} \sim \vec{H}\times\vec{v}$

for closed orbits, If surface encloses region of higher energy (hole orbit), direction is opposite than if surface encloses lower energy (electron orbit)

ex: 3-d cubic, $\hat{H} \parallel \hat{z}$ so in nearly free electron approx



$\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{km} - \delta_{jm}\delta_{lk}$

The real space orbits ($\vec{r}(t)$) can be found:

$$\vec{r}_\perp \equiv \vec{r} - \hat{H}(\hat{H}\cdot\vec{r}) \qquad \text{position in plane} \perp \text{to } H$$

$$\hat{H} \times \hbar\dot{\vec{k}} = -\frac{e\hat{H}}{c} \times (\vec{v} \times \vec{H}) = -\frac{e}{c} H (\dot{\vec{r}} - \hat{H}(\hat{H}\cdot\dot{\vec{r}}))$$

using $\vec{v} = \dot{\vec{r}}$ + vector identity

$$= -\frac{eH}{c}\dot{\vec{r}}_\perp$$

$$\text{so} \quad \vec{r}_\perp(t) - \vec{r}_\perp(0) = -\frac{\hbar c}{eH}\hat{H} \times (\vec{k}(t) - \vec{k}(0))$$

So $\vec{r}_\perp$ orbit is just $\vec{k}$ orbit rotated by ~~clockwise~~ clockwise 90° about $\vec{H}$, and scaled by $\frac{\hbar c}{eH}$

in ∥ direction

$$r_\parallel(t) = r_\parallel(0) + \int_0^t v_\parallel(t)\,dt = r_\parallel(0) + \int_0^t \frac{1}{\hbar}\frac{\partial\varepsilon}{\partial k_\parallel}\,dt$$

need not be uniform in t as $\frac{\partial}{\partial k_\parallel}\varepsilon(\vec{k})$ can vary as $k_\perp$ varies.

For spherical energy surface, we get classical result: electron moves in circular orbit $\perp$ to $\hat{H}$.
However energy surfaces need not be spherical - (when they get too near zone boundaries) - need not be closed curves! See figure 12.8 in text

~~When orbits are open, applying H can lead to~~

Motion in uniform $\perp$ $\vec{E}$ and $\vec{H}$ fields
Hall effect and magnetoresistence

$$\hbar\dot{\vec{k}} = -e\left[\vec{E} + \frac{\vec{v}}{c}(k) \times \vec{H}\right]$$

$$\Rightarrow \quad \hat{H} \times \hbar\dot{\vec{k}} = -e\,\hat{H}\times\vec{E} - \frac{eH}{c}\dot{\vec{r}}_\perp$$

$$\dot{\vec{r}}_\perp = -\frac{\hbar c}{eH}\hat{H}\times\dot{\vec{k}} + \vec{w} \qquad \vec{w} = \frac{cE}{H}(\hat{E}\times\hat{H})$$

Motion is as before, but with drift velocity $\vec{w}$ added.

To determine orbits in k space note:

$$\hbar\dot{\vec{k}} = -e\vec{E} - \frac{e}{c}\frac{1}{\hbar}\frac{\partial\mathcal{E}}{\partial\vec{k}}\times\vec{H} \qquad \text{write } \vec{E} = -(\vec{E}\times\hat{H})\times\hat{H}$$

true when $\vec{E}\perp\vec{H}$

$$= -\frac{e}{c\hbar}\left(\frac{\partial\mathcal{E}}{\partial\vec{k}} - \frac{c\hbar E}{H}\hat{E}\times\hat{H}\right)\times\vec{H}$$

$$\equiv -\frac{e}{c\hbar}\frac{\partial\bar{\mathcal{E}}}{\partial\vec{k}}\times\vec{H} \qquad \bar{\mathcal{E}} = \mathcal{E} - \hbar\vec{k}\cdot\vec{w}$$

Same as if $\vec{E}$ was absent and band structure replaced by

$$\bar{\mathcal{E}}(\vec{k}) = \mathcal{E}(\vec{k}) - \hbar\vec{k}\cdot\vec{w}$$

Orbits are intersections of surfaces of constant $\bar{\mathcal{E}}$ with planes $\perp$ to $\vec{H}$

We will assume that $-\hbar\vec{k}\cdot\vec{w}$ small enough so that if the constant $\mathcal{E}(k)$ surface is closed (open) so is the constant $\bar{\mathcal{E}}(k)$ surface. Good approx in most cases – see text for estimate of numbers.

in nearly free electron model

$\varepsilon(\vec{k}) \simeq \frac{\hbar^2}{2m}\vec{k}^2$ surface of constant energy ε is sphere of radius $\sqrt{\frac{2m\varepsilon}{\hbar^2}} = k$ in k-space

$\bar{\varepsilon}(\vec{k}) = \frac{\hbar^2\vec{k}^2}{2m} - \hbar\vec{w}\cdot\vec{k}$ surface of constant $\bar{\varepsilon}$ is given by

$$\frac{\hbar^2}{2m}\left(\vec{k} - \frac{m\vec{w}}{\hbar}\right)^2 = \bar{\varepsilon} + \frac{1}{2}mw^2$$

sphere in k-space of radius

$$k = \sqrt{\frac{2m}{\hbar^2}\left(\bar{\varepsilon} + \frac{1}{2}mw^2\right)}$$

centered about $\vec{k}_0 = m\vec{w}/\hbar$

surface of constant $\bar{\varepsilon}$ is shifted by $\vec{w}\cdot\vec{k}$ term in direction $\vec{w}$