Condensed Matter Physics

Tries to understand the macroscopic behavior of materials - ie systems with huge (often $\sim 10^{24}$) numbers of interacting particles.

Originally the field was called "solid state physics" and dealt primarily with the electrical properties of solids, ie metals, insulators, semi-conductors

While condensed matter physics has now grown to include the study of a wider variety of systems, such as liquids, polymers, complex fluids, even biological systems, the focus of this course will remain on the electrical properties of solids.

Even when one knows all the relevant microscopic interactions (for solids these are always the electromagnetic interactions that are the important ones), an exact "first principles" calculation of material properties is usually impossible. The system is just too complex to solve analytically, or even numerically (except perhaps for some smallest cases).

Example: Suppose we want to compute the properties of a metal from "first principles". We are faced with a problem of $\sim 10^{24}$ atoms consisting of nuclei and electrons, all interacting with long ranged Coulomb forces! Even if we simplify the problem to conduction electrons and ionic cores (atomic nuclei + bound inner electrons), the problem remains too complex.

For $N \sim 10^{24}$ ion cores at positions $\vec{R}_\alpha$ with momentum $\vec{P}_\alpha$, and ZN conduction electrons (Z is valence number) at positions $\vec{r}_i$ with momentum $\vec{p}_i$, the Hamiltonian is

$$\mathcal{H} = \sum_\alpha \frac{\vec{P}_\alpha^2}{2M} + \sum_i \frac{\vec{p}_i^2}{2m}$$ kinetic energy

$$+ \sum_{\alpha,\beta} V_{ion-ion}(\vec{R}_\alpha - \vec{R}_\beta)$$ ion-ion interaction

$$+ \sum_{i,j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|}$$ electron-electron interact

$$+ \sum_{i,\alpha} V_{e-ion}(\vec{r}_i - \vec{R}_\alpha)$$ electron-ion interaction

Quantum mechanically, the state of the system is described by the wavefunction

$$\psi(\vec{r}_1, s_1, \vec{r}_2, s_2, \cdots, \vec{R}_1, S_1, \vec{R}_2, S_2, \cdots)$$

where s_i and S_α are the spins of electron i and ion α, respectively.

We then need to solve for the eigenstates

$$\mathcal{H}\psi_n = E_n\psi_n \qquad \text{where } \psi \text{ has } \sim 10^{26} \text{ degrees of freedom}$$

To compute properties at finite temperature, we would then take these eigenstates and construct the partition function

$$Z = \sum_n e^{-E_n/k_BT}$$

Clearly this plan of solution is impossible to carry out!

Progress in condensed matter physics is therefore a process of making simplifying assumptions that allow one to do a calculation, then checking the result against experiment to see if there is agreement. When new and unexplained experimental data comes up, the goal is then to see if it can be explained by doing a better calculation within the previous accepted approximations, or whether the new data requires one to relax some approximation and include a previously ignored physical process - then the challenge is in understanding what is the relevant physical process or mechanism that

is responsible for the new effects, and how to make an approximation for this mechanism that will allow one to do a new and better calculation.

Standard approximations to start

1) Assume ions are arranged in fixed positions and cannot move - "static lattice approx"

2) Assume electrons do not interact with each other - "independent electron approx"

3) Assume electrons do not interact with the ions - "free electron approx"

What we are left with after all these approx's is an ideal gas of conduction electrons. We effectively assume that all the e-e or e-ion interactions do is to give instantaneous collisions that serve to help the electron gas at some fixed temperature T.

Surprisingly, one can explain a lot about metals using this very drastic single model. We will therefore spend the first part of the course dealing with this free electron model.

Drude Model (~1900)

Classical ideal gas of free electrons to describe behavior of conduction electrons in a metal

What is a metal? Atoms of atomic number Z_a with valence Z. From each atom, Z electrons "detach" from the ionic core and are free to wander through the material. These are the conduction electrons (so each ion remains with $Z_a - Z$ bound core electrons)

N_a is number of atoms

$N = ZN_a$ is number of conduction electrons

conduction electron density in volume V

$$n = \frac{N}{V} = 0.6022 \times 10^{24} \frac{Z\rho}{A}$$

↑ Avogadro's number

ρ = mass density of metal (g/cm^3)

A = atomic mass of atom ($g/mole$)

Another measure of electron density is

r_s = radius of sphere whose volume is the volume per conduction electron

$$\frac{4}{3}\pi r_s^3 = \frac{V}{N} = \frac{1}{n} \quad \Rightarrow \quad r_s = \left(\frac{3}{4\pi n}\right)^{1/3}$$

Convenient to give r_s in units of the Bohr radius $a_0 = 0.529 \times 10^{-8}$ cm

Some metals

Element	Z	$n (\times 10^{22}/cm^3)$	r_s (Å)
Li	1	4.7	1.72
Na	1	2.65	2.08
K	1	1.40	2.57
Rb	1	1.15	2.75
Cs	1	0.91	2.98
Ca	1	8.47	1.41
Ag	1	5.86	1.60
Au	1	5.90	1.59
Be	2	24.7	0.99
Mg	2	8.61	1.41
Ca	2	4.61	1.73
Sr	2	3.55	1.89
Ba	2	3.15	1.96

generally r_s ~ 2 – 3 Å