Drude model

Conduction electron obeys equation of motion:

$$\frac{d\vec{p}}{dt} = \vec{f}_{ex}(t) + \vec{f}_{coll}(t)$$

$\vec{f}_{ex}$ is external applied force, typically from applied electric and magnetic fields,

$\vec{f}_{coll}$ is force due to "collisions" – imagined to be collisions between the electron and either the ions or other electrons (both these turn out to be wrong)

Assume collisions are <u>instantaneous</u> – ie time for a collision is $\ll$ time between collisions (somehow long range coulomb forces are being replaced by short ranged "collisions"!)

Between collisions, electrons moves like a free particle acted on by force $\vec{f}_{ex}$.

Collisions are instantaneous events that change electron velocity randomly – serve to keep electron gas in thermal equilibrium

Collision occur at rate $1/\tau$

τ = "<u>relaxation time</u>"

Average equation of motion over all electrons

$$\langle \vec{p} \rangle = \frac{1}{N} \sum_i \vec{p}_i$$

$$\frac{d\langle \vec{p} \rangle}{dt} = \vec{f}_{ext}(t) + \langle \vec{f}_{coll} \rangle$$

What is $\langle \vec{f}_{coll} \rangle$?

Electron enters collision with momentum $\vec{p}_{initial}$.
Electron exits collision with momentum $\vec{p}_{final}$
Change in momentum is $\vec{p}_{final} - \vec{p}_{initial}$
Rate of collisions is $1/\tau$

So: $$\langle \vec{f}_{coll} \rangle = \frac{\langle \vec{p}_{final} - \vec{p}_{initial} \rangle}{\tau}$$

Now by assumption, the electron exists a collision with random momentum chosen from the equilibrium distribution. $\Rightarrow \langle \vec{p}_{final} \rangle = 0$
(since equilib distrib depends only on $|\vec{p}|$)

$$\langle \vec{f}_{coll} \rangle = - \frac{\langle \vec{p}_{initial} \rangle}{\tau}$$

But $\langle \vec{p}_{initial} \rangle = \langle \vec{p} \rangle$ average momentum

So finally $\langle \vec{f}_{coll} \rangle = - \frac{\langle \vec{p} \rangle}{\tau}$ and

$$\boxed{\frac{d\langle \vec{p} \rangle}{dt} = \vec{f}_{ex}(t) - \frac{\langle \vec{p} \rangle}{\tau}}$$

Hence forth we drop the brackets <>, and $\vec{p}$ will be used to denote the <u>average</u> momentum of the electrons

<u>DC electric conductivity</u>

<u>Ohms Law</u>: $\underset{\text{voltage}}{V} = \underset{\text{current}}{I}\,\underset{\text{resistance}}{R}$

$V = EL$, E is electric field, L is length

$I = jA$, $\vec{j}$ is current density, A is crossection area

$$\Rightarrow E = j\frac{AR}{L} = j\rho$$

where $R = \frac{L\rho}{A}$, ρ is the <u>resistivity</u>

$j = \left(\frac{1}{\rho}\right)E = \sigma E$, σ is the conductivity

Expect ρ, σ to be independent of system volume

<u>Compute σ within Drude model</u>:

$\vec{f}_{ex} = -e\vec{E}$ $\quad$ $-e$ is electron charge

$\vec{E}$ is uniform dc electric field

$$\frac{d\vec{p}}{dt} = \vec{f}_{ex} - \frac{\vec{p}}{\tau} = -e\vec{E} - \frac{\vec{p}}{\tau}$$

for steady state d.c. behavior, average momentum is constant in time, so $d\vec{p}/dt = 0$

$$\Rightarrow \quad \vec{p} = -e\vec{E}\tau$$

Current density $\vec{j} = -ne\vec{v}$

$\vec{v} = \frac{\vec{p}}{m}$ is average electron velocity

n is electron density

$$\vec{j} = -ne\frac{\vec{p}}{m} = -\frac{ne}{m}(-e\vec{E}\tau) = \frac{ne^2\tau}{m}\vec{E}$$

$$\rightarrow \quad \boxed{\sigma = \frac{ne^2\tau}{m}}$$

Since the collisions are just treated phenomenologically in the Drude model, rather than coming from a more microscopic theory that would allow one to compute τ, the above is really just a way to determine Drudes relaxation time τ from experimental measurement of conductivity σ.

Typical resistivities ρ at room temperature are

$\rho \sim 10\ \mu\Omega$-cm (micro Ohm-cm)

gives $\tau \sim 10^{-14} - 10^{-15}$ sec

Is this a reasonable value for τ?

mean free path = average distance between collisions

$$\ell = v_0 \tau$$

$\uparrow$ average electron speed

For a classical ideal gas of electrons

$$\langle \tfrac{1}{2} m v_0^2 \rangle = \tfrac{3}{2} k_B T$$

gives

$v_0 \sim 10^7$ cm/sec at room temperature

using $\tau \sim 10^{-14}$ to 10^{-15} sec at room temp

gives $\ell \sim 1$ to 10 Å

Above value of ℓ is comparable to inter ionic spacings, it seems that Drude model is consistent with idea that collisions are due to electrons colliding with ions.

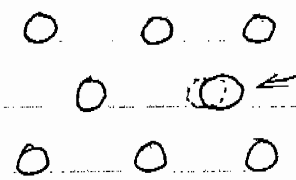
But: at lower $T \sim 77°K$, v_0 is smaller since $v_0 \sim \sqrt{T}$, but ρ is measured to be smaller too!

at $T \sim 77°K$, $\rho \sim 1\ \mu\Omega$-cm $\Rightarrow \tau \sim 10^{-13}$ to 10^{-14} sec

$v_0 \sim .5 \times 10^7$ cm/sec $\Rightarrow \ell \sim 5$ to 50 Å

When we consider the more correct quantum mechanical treatment of the electron gas, we will find that actually $v_0 \sim 10^8$ cm/sec and is roughly independent of temperature, thus giving $\ell \sim 100$ to 1000 Å at low $T \sim 77°K$. This is too big to be explained by a simple picture of colliding with static ions!

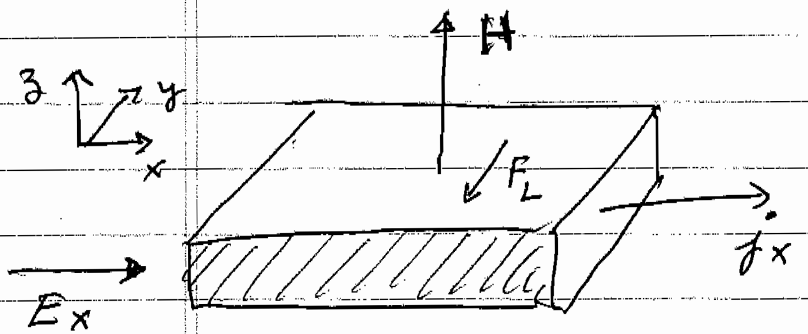
In quantum picture, ~~we will~~ where electron is viewed like a wave, we will see that a periodic array of static ions does not, in fact, scatter the electron at all! Collisions are therefore not with the ions, but rather occur when an ion is displaced from its periodic array position due to thermal vibrations. These thermal vibrations of the ions are "phonons". Electric resistivity is thus due (except at very low temp) due to "electron-phonon" scattering.



← ion displaced from its perfect periodic position causes scattering of electron wave.

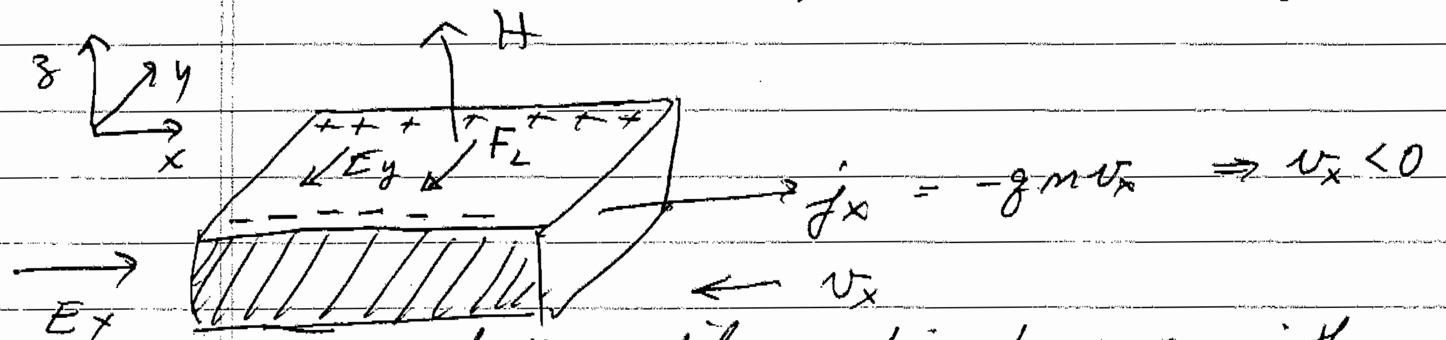
Hall effect (1879) - determines the sign of the charges that carry the electric current in a metal

electron motion in combined static electric and magnetic fields



Electric field E_x applied in $\hat{x}$ direction produces flowing electric current j_x in $\hat{x}$ direction. Magnetic field H in $\hat{z}$ direction exerts Lorentz force $\frac{\vec{j}}{c}\times\vec{H}$ on the ~~charges carr~~ moving charges carrying the current $\vec{j}$. For $\vec{j}$ in $\hat{x}$ direction and $\hat{H}$ in $\hat{z}$ direction, this Lorentz force $\vec{F}_L$ is in the $-\hat{y}$ direction. $\vec{F}_L$ deflects the charge carriers to the side wall of the wire (the shaded wall in the figure) where they build up and create a surface charge density. The surface charge density produces an electric field E_y in $\hat{y}$ direction. For a steady state situation, the force from E_y will exactly cancel out the Lorentz force F_L. If W is the width of the wire, then measuring the "Hall voltage" $V_y = E_y W$ allows one to determine the sign of the charges that carry the electric current.

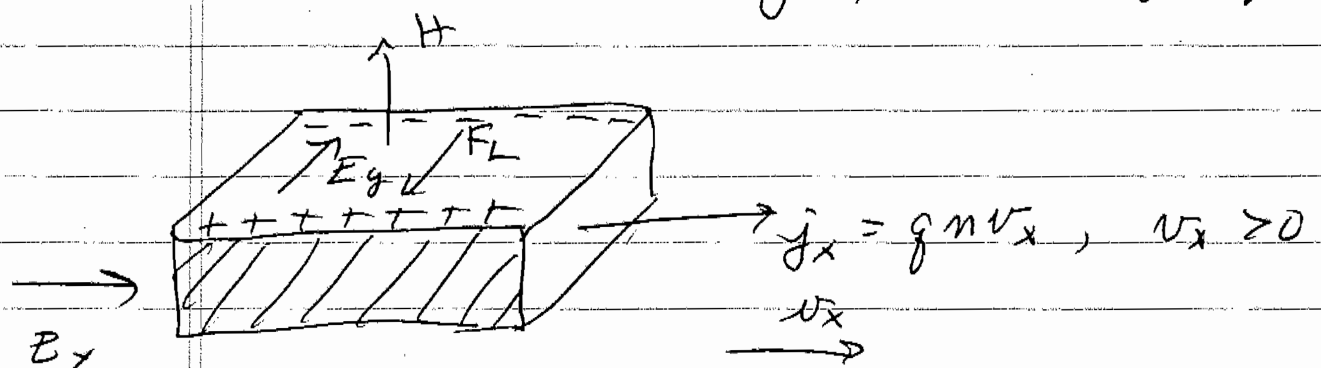
If current is carried by negative charges $-q$, then



F_L deflects the mobile negative charges carrying the current, and negative charges build up on shaded surface (neutrality of system ⇒ absence of negative charge, i.e. positive charge, builds up on opposite surface)

The electric field E_y is in $-\hat{y}$ direction and Hall voltage is <u>negative</u>

If current is carried by positive charges $+q$, then



F_L deflects the mobile positive charges carrying the current and positive charge builds up on the shaded surface

The electric field E_y is in the $+\hat{y}$ direction and the Hall voltage is <u>positive</u>

For most (but not all) metals one finds a negative Hall voltage. This established that it was negatively charged electrons that carry the electric current in most metals.

Quantities to measure

Hall coefficient $R_H \equiv \dfrac{E_y}{j_x H}$

since we expect force from E_y to exactly balance out Lorentz force F_L in steady state, we expect R_H to be independent of H

magnetoresistance $\rho(H) \equiv \dfrac{E_x}{j_x}$

We can compute both R_H and ρ using the Drude model.

$$\frac{d\vec{p}}{dt} = -e\left(\vec{E} + \frac{\vec{p}}{mc} \times \vec{H}\right) - \frac{\vec{p}}{\tau} = 0 \text{ in steady state}$$

for x and y components

$$0 = -eE_x - \frac{eH}{mc} p_y - \frac{p_x}{\tau}$$

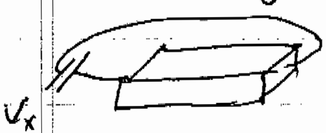
$$0 = -eE_y + \frac{eH}{mc} p_x - \frac{p_y}{\tau}$$

$\omega_c \equiv \dfrac{eH}{mc}$ cyclotron frequency = angular frequency of a charged particle in circular motion in uniform H

① $eE_x = -\omega_c p_y - \frac{p_x}{\tau}$

② $eE_y = \omega_c p_x - \frac{p_y}{\tau}$

In steady state, current flows only in $\hat{x}$ direction.



No current flows out the side walls of the wire $\Rightarrow p_y = 0$

with $p_y = 0$,

① $\Rightarrow \quad p_x = -eE_x\tau$

$$j_x = -nev_x = -\frac{nep_x}{m} = \frac{ne^2\tau}{m}E_x$$

$$\boxed{\frac{E_x}{j_x} = \frac{m}{ne^2\tau} = \rho}$$

magnetoresistance $\rho(H) = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$

same as ordinary d.c. resistivity ρ when $H=0$

In Drude model, $\rho(H)$ is independent of H!

agreed with expt measurements by Drude.

More modern expts however do find ρ can vary with H.

② $\Rightarrow E_y = \frac{\omega_c}{e}p_x = -\omega_c\tau E_x$

Hall coefficient $R_H = \frac{E_y}{j_x H} = \frac{\left(\frac{\omega_c}{e}p_x\right)}{\left(-\frac{nep_x}{m}\right)H} = \frac{-\omega_c m}{ne^2 H}$

use $\omega_c = \frac{eH}{mc} \Rightarrow R_H = \frac{-\left(\frac{eH}{mc}\right)m}{ne^2 H} = -\frac{1}{nec}$

$$\boxed{R_H = -\frac{1}{nec}}$$ Hall coefficient independent of H

But also, R_H is independent of our phenomenological parameter τ, the relaxation time.

R_H is something we can directly test against experiment since it only depends on the electron density n, which can be easily calculated.

In practice R_H is found to depend on H and T and also on sample preparation. But at low T, high H ($\sim 10^4$ g) very pure samples, R_H is found to approach a constant value, often very close to the Drude value

	metal	valence	$-1/R_H nec$ (=1 for Drude)	
alkalis	Li	1	0.8	Drude prediction very good for alkali metals which have single s shell electron as valence electron
	Na	1	1.2	
	K	1	1.1	
	Rb	1	1.0	
	Cs	1	0.9	
	Cu	1	1.5	
	Ag	1	1.3	
	Au	1	1.5	
	Be	2	−0.2	sign is negative! ⇒ current is carried by objects with positive sign
	Mg	2	−0.4	
	In	3	−0.3	
	Al	3	−0.3	