PHY404 Linear Spaces Spring 2006

Final Exam Wednesday May 3 2006 9:30-10:45am

1 Define the creation-annihilation operators by

$$a^\dagger |n> = \sqrt{(n+1)}|n+1> \text{ for } n = 0, 1, \cdots, \quad a|n> = \sqrt{n}|n-1> \text{ for } n = 1, 2 \cdots, \tag{1}$$

and $a|0> = 0$. Here the collection of vectors $|n>, n = 0, 1, \cdots$ is an orthonormal basis.

1.1 For any complex number z, find an eigenvector for a with eigenvalue z, as a linear combination $\sum_{n=0}^{\infty} c_n(z)|n>$.

1.2 Find the length of the eigenvector by evaluating the sum $\sum_0^{\infty} |c_n(z)|^2$ and use that to find an eigenvector of unit length.

2 Consider the matrix

$$A = \begin{pmatrix} 1 & -i & 0 \\ i & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{2}$$

2.1 What are its eigenvalues and eigenvectors?

2.2 Find its resolvent $R(z) = (A - z)^{-1}$ and verify that the positions of its poles are the eigenvalues of A.