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Rosseland and Planck mean opacity

Amy Zou

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1 INTRODUCTION

2 PLANCK OPACITY

The weighting is the normalized Planck black body radiation distribution.

$$\kappa_{\rm Pl} = \frac{\int_0^\infty \kappa_\nu \, B_\nu(T) {\rm d}\nu}{\int_0^\infty B_\nu(T) {\rm d}\nu} = \left(\frac{\pi}{\sigma_{\rm SB} T^4} \right) \int_0^\infty \kappa_\nu \, B_\nu(T) {\rm d}\nu \tag{1}$$

Amy: maybe a factor of 2 missing on the RHS above. Since Stefan–Boltzmann law is the Planck function integrated over half a sphere:

$$\int_0^\infty {\rm d}\nu \int_\cap {\rm d}\Omega \, B_\nu \cos\theta = \int_0^\infty {\rm d}\nu \int_0^{\pi/2} {\rm d}\theta \int_0^{2\pi} B_\nu \cos\theta \sin\theta = \sigma_{\rm SB} T^4$$

, and $\int_0^{\pi/2} \sin\theta {\rm d}\theta \int_0^{2\pi} {\rm d}\phi = 2\pi$ *.*

3 ROSSELAND OPACITY

Use a temperature derivative of the Planck distribution

$u(\nu, T) = \partial \, B_\nu(T) / \partial T$

as the weighting function.

$$\frac{1}{\kappa_{\rm Rs}} = \frac{\int_0^\infty \kappa_\nu^{-1} u(\nu, T) {\rm d}\nu}{\int_0^\infty u(\nu, T) {\rm d}\nu} \tag{2}$$

This is derived in the Diffusion Approximation to the radiative transport equation.

Valid when radiation field is isotropic over scale that

$$l \lesssim l_{\rm mfp} = \frac{1}{n \sigma_\nu} = \frac{1}{\rho \, \kappa_\nu}$$

3.1 mean opacity for Thomson electron scattering

$$\kappa_{\rm es]} = 0.20 (1 + X) \, {\rm cm}^2 {\rm g}^{-1} \tag{3}$$

where X is the hydrogen mass fraction.

3.2 Approximation from Collins chapter 4

An approximation form of the Rosseland mean opacity, independent of frequency. It's on page 84 of the e-book (p. 8 of the chapter 4 PDF file). Form (a) will be enough if we assume Bremsstrahlung only as a first approach. The only task left is to have a starting point $\bar{\kappa}_0$ for the gas composition we are using, which depends on the chemical composition.

$$\bar{\kappa} = \bar{\kappa}_0 \rho^n T^{-s} \tag{4}$$

(a)	$n = 1$	$s = 3.5$	Krammers' law
(b)	$n = 0.75$	$s = 3.5$	Schwarzschild's opacity
(c)	$n = 0$	$s = 0$	electron scattering

4 OTHER SOURCES

- Iglesias & Rogers (1996) OPAL opacity tables. Temperature range: $3.75 < \log T < 8.7$.
- Collins (1989) provides approximation forms of the Rosseland mean. Also the textbook used for Eric's stellar astrophysics course. `http://ads.harvard.edu/books/1989fsa..book/`
- Semenov et al. (2003) Gives opacity table for temperature under 10^3 K, in protoplanetary disk.
- Mayer & Duschl (2005) still lower temperature region for the primordial matter. *Amy: See Table 4 therein for a comparison of several tabular opacities. The one we will probably use in the future is OPAL.*
- Tsakiris & Eidmann (1987). Old theory paper, probably won't go that deep for now.
- Cox et al. (1991). Another old textbook, page 1194-1195.
- Jiang et al. (2016) is a recent numerical work. One of the author, Shane Davis, from UVA, is a possible contact about the problem.
- Cox & Tabor (1976) one of the very early work on this.

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